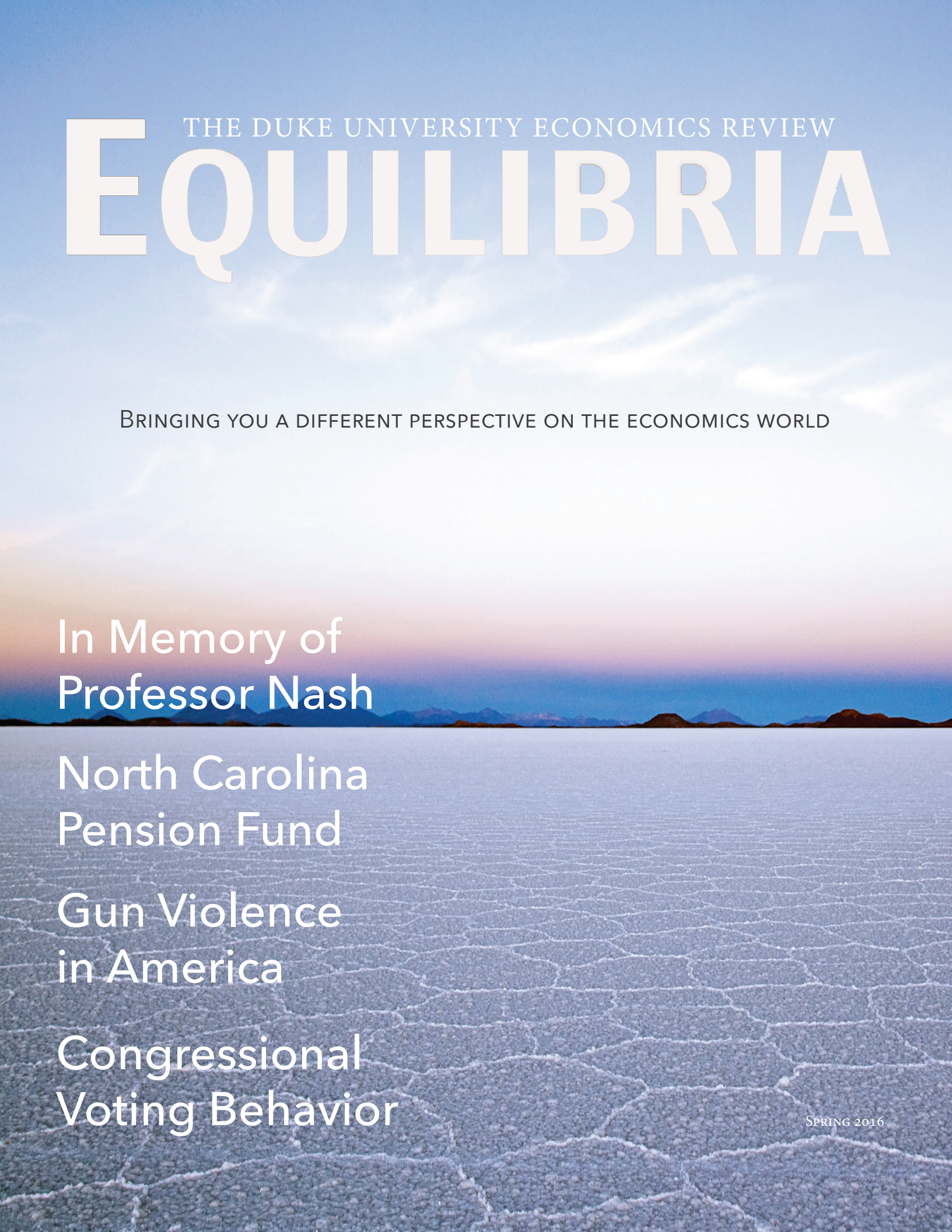

THE DUKE UNIVERSITY ECONOMICS REVIEW

EQUILIBRIA

BRINGING YOU A DIFFERENT PERSPECTIVE ON THE ECONOMICS WORLD

In Memory of Professor Nash

North Carolina Pension Fund

Gun Violence in America

Congressional Voting Behavior

SPRING 2016

EDITOR'S NOTE

Economics is not all about mathematics and models. Many Economics students in colleges go through the grueling Microeconomics, Macroeconomics and Econometrics curriculum to be trained as fledgling economists. While many students like myself are engrossed with mastering the textbook materials, we often overlook how fun and relevant Economics can be to current affairs and our everyday lives. This is why our team of editors at Equilibria seek to bring you a different perspective on Economics. This edition publishes the newest opinion on topics relevant to congressional voting, Professor John Nash's legacy, gun violence, and the ongoing pension fund management strategy in North Carolina. As Equilibria becomes officially chartered with Duke Student Government as Duke's Undergraduate Economics Journal and welcome Professor Edward Tower on board as our faculty adviser this semester, we hope to continue bridging the gap between fresh research insights and students in years to come.

Sincerely,

Jackie Xiao

DUKE ECONOMICS REVIEW

Equilibria: The Duke University Economics Review is the official publication of the Economics Student Union at Duke University. This magazine is dedicated to presenting select research and opinions by university scholars on all fields relevant to economics.

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We would like to thank the Duke University Department of Economics, the Duke Economics Student Union, contributing Professor Edward Tower and all student writers.

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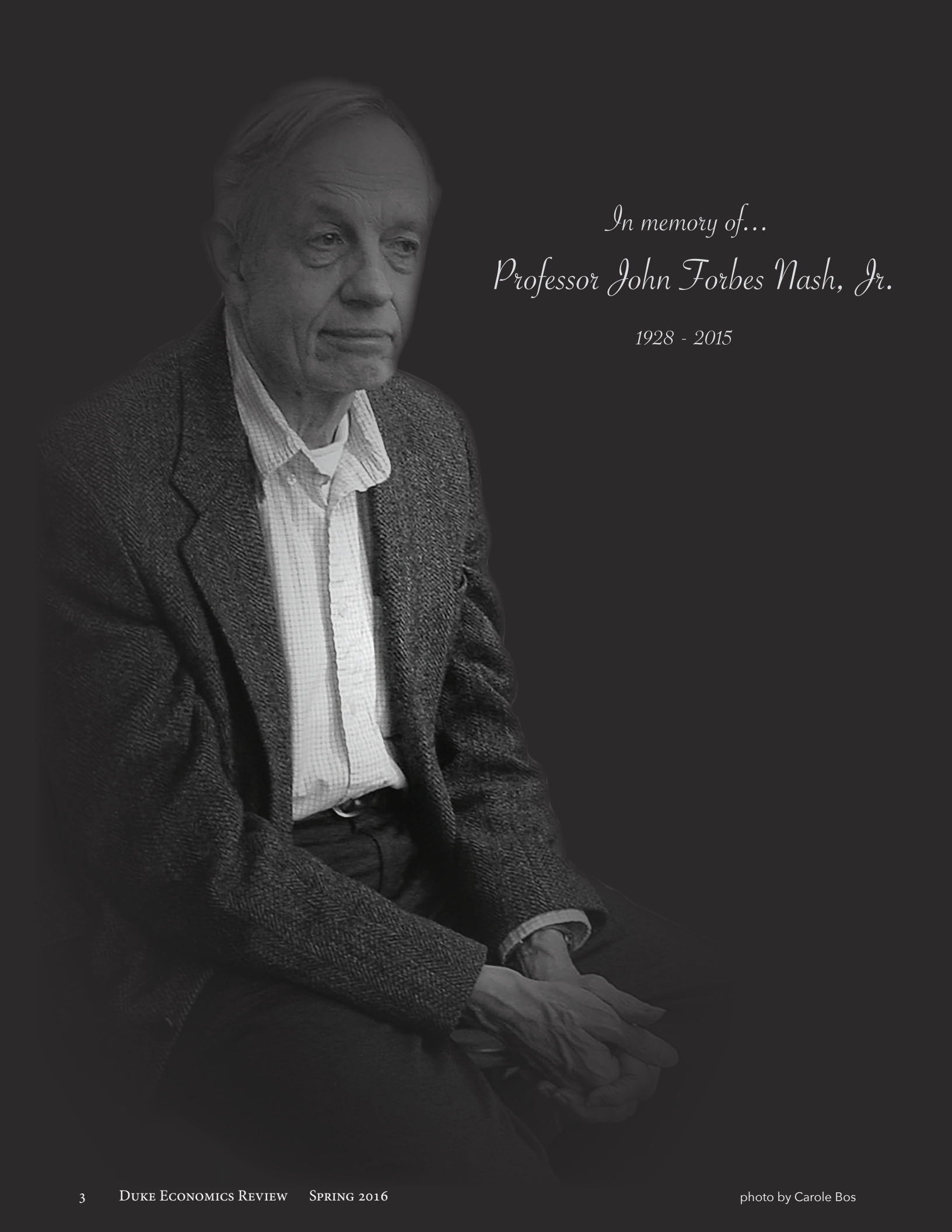


In memory of...

Professor John Forbes Nash, Jr.

1928 - 2015

photo by Carole Bos

From a Legend in Letters to a Living Inspiration

My Impression of Professor Dr. John Forbes Nash Jr.

Wasutadon Arm Nakawiroj, Chulalongkorn University

Editing by
Marisa Witayananun and Jackie Xiao

At a small, beautiful island in the southernmost part of Germany, in the picturesque Lake Constance, an inspiring conference is held every four years. Prominent scholars from the disciplines of Medicine, Science, and Economics are invited to give lectures to a group of "young scientists" selected from various countries. This is the Lindau Nobel Laureates Meeting. Attended by many prominent Nobel Laureates and the aspiring young researchers who expect to meet and interact with their academic role models, the event continually receives attention from many nations and institutions. The full name of the event was "The Fourth Lindau Nobel Laureates' Meeting in Economic Sciences", or Lindau 2011, in short. This is where it all took place.

Our travel and logistics arrangements were smooth and were progressing steadily, thanks to the wonderful and professional staff crew at the Lindau Nobel Foundation, who attended with care the requests and specifications of each participant. This story, this article, may never be considered complete without the proper thanks to the people who made it all possible. As the logistics and supporting details were seamless, we young researchers were able to focus our fullest on the events and lectures. For me, it was a historic event. As a young Ph.D. candidate fresh out of the classroom, I was inspired to meet the intellectual giants of our profession, the greatest minds of modern economics. I remember the turning point of my life in my last year at high school, which began with a book on the life and work of Nobel Laureates in Economics, which eventually culminated in my decision to apply for economics as a university degree instead of my previous decision to be an engineer. In the eyes of 17-year old economists-to-be, I and most students who apply for economics were looking at our professors and the full-fledged economists with great admiration, and this sentiment persisted many years from that day. That was my part, my story, and I am certain that my friends and peers at the conference all felt that way. For us, it was like the first day we got admitted, where we were both excited and thrilled to meet the professors, the real experts in the field. The meeting at Lindau reminded us the atmosphere we had at the orientation day back in Bachelor's degree – that we were becoming part of something wonderful, and that we get to interact with our academic role models. That's the impression we got when we joined the event, and it is what we wish for the younger generation to have upon joining academia – it is, simply put, passion.

Each of the 18 Laureates at the Lindau Nobel Laureates Meeting of 2011 (Lindau 2011 for the sake of brevity) deserves great recognition and high regard for their contributions to modern economics, at any rate, I got the opportunity to spend the most time with Professor Nash amongst all the Nobel Laureates of that meeting. I know full well that there are many others who have spent far more time with him, who know a great deal about his life and work than I do, and they will be able to tell you about John Nash's great achievements much better than I am able to. So, and at any rate, given my opportunity to meet him and the enthusiasm to write accounts of the memory, this article will share with you a glimpse of a young Ph.D. candidate's impression of John Nash at his honorary sessions and stay at Lindau. The contents of this article are drawn from my chance to meet Dr. Nash and Madame Alicia L. Nash, his loving wife, on numerous different occasions – the plenary lecture, the parallel lecture, the official dinner, the boat trip in Lake Constance, the garden tour on Mainau Island, and, as an unexpected opportunity, the fateful occasion to meet him and his wife on the day after the conclusion of the meeting, where we took a walk across town to the taxicab station. Of course, most people have heard about John Nash as a human supercomputer and a brilliant mathematical mind (which is undoubtedly was), whereas, as my own way of honoring his memories, this article will provide another side of him as an amiable human being. I am certain that my Lindau 2011 friends also have lots of their own stories with their favourite laureates to tell you - something I'd love to read.

To begin with, we, the young researchers selected to participate in the Fourth Lindau Nobel Laureates' Meeting of 2011, were all excited to know that the Nobel

Laureates were to be present at the event. I remember the moment we all sat at our seats and each of us turned around every now and then to see whether the laureates had arrived. The nature of the meeting was also wonderfully arranged to provide and promote interactions between the laureates and the young researchers. It was where great economists of the highest caliber came to give speeches and presentations of their latest research, discuss current global issues, give lectures and advice to distinguished guests, honorable scholars from around the globe, and also to us young researchers as their young, much less experienced counterparts. The laureates even got to join us in casual events & informal interactions. I am usually not the type to get excited when meeting pop culture celebrities or movie stars, but the presence of the Nobel laureates was something else. It is not an emotional craze or a trending fad coming from charming, glamorous appearances, but they inspire us through the thought that we can create something new, something better, something useful, for the world. Most importantly, they have a message to tell. They have the knowledge to teach. Being in that professional atmosphere reminds us that, if we try hard enough, there will be a way through which we can make a difference in the world, through research and clever inquiry of the facts and figures. We were hoping that one day our curiosity would be able to make good things in the world, what people can learn from and expand on to benefit the society in ways we have yet to know.

Cameras and mobile phones were out. Flashes were turned off and hands were raised to silently but quickly take photos of our favourite laureates, since this is a very rare opportunity to meet them in actual person, as opposed than through television or in books and magazines. I got lucky and took a clear and valuable picture of Professor Christopher Pissarides, one of the winners of the 2010 Nobel Prize in Economics, just before we immediately noticed Professor John Nash walking down the hall right to the spot that I had been taking pictures of. We, the young economists, were very excited (and were taking pictures too, of course). That marked the beginning of the story that we of in Lindau 2011 will always treasure. The first day was focused on the other laureates – the opportunity to listen to his presentation and ideas came on the second day.

On the second day, following the wonderful presentations and discussions by several other laureates, Professor Nash gave his plenary lecture on the topic of "Ideal Money". He appeared in a light brown suit and was calmly and thoughtfully walking across the stage during his presentation. I was in the frontmost row and thus was able to see him clearly in person, in addition to his projected image on the enormous screen. I happen to notice that he was one of the Laureates who were scheduled to give one of the parallel lectures on the afternoon of the same day.

His talk revolved around the different concepts of money and how these factors play a role in the economy. One of his remarks early on in the lecture was that rationality comes into play when money is regarded, as people are influenced by the characteristics of money to the degree that their rationality in making decisions are frequently compromised. Another remark was that in traditional views money are often associated with evil and malice, and his argument against this was based on how money can also be used for good purposes, for example, donations that are made in terms of money are likely to be more useful than when donations are made in other commodity forms. He added that money is a beneficial and useful component in a world where resources are finite, as the condition of finite resource in turn leads to the need of exchanges and transactions.

He proposed that "honesty is the best policy" when it comes to monetary matters and handling inflation, and that he supported a zero target rate for inflation. One of his conclusions to the talk was that reliable estimates for future values of a currency benefit businesses and facilitate contractual agreements.

Another highlight of the Lindau program was the parallel lecture sessions, which quickly prompted us the concept of opportunity costs; we had to select one of the talks held at the same time, so we were assessing our preferences on academic interest. Central bankers would often elect to go to monetary policy and macroeconomics sessions, while academics tend to gravitate towards topics concerned with theoretical curiosities such as microeconomics or human behaviour. These parallel sessions by the Laureates are thought-inspiring and awe-inspiring at the same time; young researchers get to listen to their wisdom like in a conventional discussion-based classroom, to ask questions, and to meet the Laureates up-close. On the first day, I went to the parallel lecture held by Professor Dale T. Mortensen, one of the three joint Nobel Prize winners of 2010.

The Lindau Meeting allows participants to make visits to various landmarks and historical attractions on the island, as certain sessions are held separately from the main event. The venue of the lecture was located at some distance from the Inselhalle (The Island's Convention Hall), and we had to make sure that we arrived on time, as we would not want to miss any of the events.

The interior of the building was filled with cultural and artistic decorations. John Nash was there on the stage, sitting quietly. No one did make an explicit comment on this, but I felt that Professor Nash's calm presence alone was enough to put the stage into an atmosphere of elegance, as if a live performance was underway. In a sense, for a young economist like me and my fellow researchers, he commanded an aura of awe comparable to a live musical performance.

However, his lecture was filled with the air of serenity and there was gentleness coming from his words as he reflected his past achievements and progress in the part of the lecture that were meant as a career and academic advice for the young researchers. He was addressing his experiences and was using them as part of the lessons for us, perhaps from seeing that a lot of us wanted to know what it was like to be the John Nash, and what his earlier life was like. He was sitting on the chair on the stage and we were all looking up to him, both literally and figuratively! It was magic - we, young economists from around the world, were actually attending one of Professor Nash's lectures. For the entirety of the session, he sat on the chair in a dignified, yet benign manner.

Within this particular session, I got the opportunity to ask one question about "modern international trade from a game theorist's point of view", and after receiving his opinions and writing them down, another question soon crossed my mind. When there was silence again in the Q&A session, I took the courage to propose the second question regarding how people, as players in real world "games", could escape the mutually inferior outcome in a situation known as "Prisoners' Dilemma". His replies, amongst other things, mentioned one of his then recent papers, published in a journal specializing in Game Theory.

After the parallel lecture, we were informed that we were to attend the official dinners, where young economists were grouped and assigned to the various restaurants located on the island. Several Nobel Laureates would then be randomly selected to join us in the dinner. The one that I was assigned to was named "Wissingers", a restaurant with a nice and cozy dining terrace located in a garden. When I reached the restaurant, two long tables were in place, with Professor Roger Myerson (whose wonderful textbook on game theory I used for my Doctoral Microeconomics Course) at one of the tables, with many young researchers positioned around him. As I began to wonder who the second Laureate could have been, I saw Professor Nash, whom I had just met at the parallel lecture, walking towards the second table. Delighted, I walked up to the Professor, re-introduced myself in a more informal manner (compared to the brief introduction made at the previous section). He seemed to have recalled me, too, perhaps as the result of my questions back in the latter half of the lecture (which was no further than 2 hours before the dinner). He then took a seat located between two other vacant seats. I sat down on the seat immediately to his right and we began to talk about many things. The topics quickly became informal and casual ones, befitting the beautiful evening at the restaurant.

The succeeding conversations were lively and highly memorable, especially with everyone sitting together and exchanging our ideas, comments, questions, and even casual topics. At that very long table, Madame Alicia's seat was exactly facing the Professor's, and we, the young researchers, were lucky to sit around the Professor and his wife . Questions and curiosities started to pour in from each of us, and the topics started with some academic and technical issues but quickly moved on to casual remarks.

The conversation went on for hours – from around 6pm to as late as 10pm. What seemed to have started as a formal and official dinner quickly evolved into lively and flavorful discussions of topics of various natures. I continued to express my curiosity on game theory and cooperation, which led to one of the two questions that I asked him back in the parallel lecture. After some explanation of his opinions, he wrote for me on the restaurant pamphlet the name and address of one of his papers corresponding to that curiosity . In addition, having brought my "Lindau Yearbook" with me on that dinner, I asked him whether he could sign the book for me. To my delight, the following photo came to exist.

Later on in that conversation, approximately around 9 and 10pm, the night sky of Lindau was littered with stars. A friend named Roy, from India, came up with a question. In the movie "A Beautiful Mind", Professor Nash had picked out a pattern from the night stars. My friend's curiosity was about how much, or whether, that scene was true. In response to my friend's question, he replied that he could have done the feat in question if he would, but that the particular scene represented in the movie did not actually occur. Saying that several details of his life had been exaggerated in the movie, understandably for thought-inspiring purposes, Professor Nash then gave a peaceful smile.

The Professor had also joined us on the boat trip to the isle of Mainau, along with several other laureates. On the island, everyone traveled in different directions, as the island's park covered a vast tract of land. Around five of my friends and I came across Professor Nash on the island, as we were viewing the livestock section of the island. We had a casual exchange of ideas of how and why cows of some species have horns and some don't. Afterwards, the Professor and we then proceeded through the botanical sections of the park and came across the Scientific Discoveries in Health Exhibition. After a long walk, we went back to the Island's castle for the closing ceremony, followed by a touching and memorable evening boat trip back to the town of Lindau.

After the boat trip and the closing ceremony, the Meeting was wonderfully concluded and everyone was heading home. Like several others, my flight was scheduled several days after the day of the closing ceremony, so I decided to pay a visit to the Lindau Nobel Foundation's office, located next to the town's train station, which was in turn adjacent to the taxicab station. The bottom floor of the building houses the town's tourism office, and is the ideal place to browse for information on tourist attractions located throughout the city. And after a pleasant talk with the ever-diligent, ever-dedicated officers at the Foundation, I went down the stairs and back to the first floor . To my joy and surprise, as I opened the door leading back to the tourism office and the building entrance, Professor Nash and Madame Alicia were standing right there! After another warm greeting, I told the Professor that my flight was scheduled on the next day, so I came to pay a visit to the Foundation headquarters, and he told me that his initial flight was postponed due to the storm that had occurred the day before. As a result, he decided to spend a few more days in Lindau and enjoy the town's beautiful scenery. The conversation on that day was of a unique and warm atmosphere. Professor Nash, Madame Alicia and I were standing at the beautifully decorated shelf section of the tourism office, full of nice brochures and pamphlets on each of the town's attractions, and we talked merrily about the nice places that each of us should go and see or revisit.

As we said goodbye, I thought it was only appropriate to do the "Waii", which is a normal custom in my country, where individuals put their hands together (like when one prays) and bows their head as a sign of respect to the person that they are speaking to (in Thailand, students are expected to perform this act first to their teachers and professors as a gesture of respect, where teachers reciprocate in a more reserved manner as a gesture of kindness towards the younger individuals) . Again, to my joy and surprise, the Professor reciprocated the Waii in a graceful and dignified manner, with a smile that I will never forget. Anyone who happens to be in the same situa-

tion could easily see the tranquility and benevolence in his eyes. I left the building and went into the streets happily, as the Professor and Madame Alicia stayed on and browsed through the list of the town's great attractions.

The streets of Lindau seemed to be a bit different after the big event. The crowd of academics, scholars and young researchers who participated seemed to have lightened up, and families with small kids and energetic tourist groups on vacation filled the gaps in their absence. Street performers of different art genres were a common sight as well. Noting these changes, I walked through the streets of Lindau for a great deal of time, thinking back to the great collection of events that had occurred during the week before. Yet again, to my surprise, I came across Professor Nash and Madame Alicia once more on the main street. The Professor asked me whether I would be able to accompany him and his wife to the town's taxicab station. Without hesitation, I joined the couple on that afternoon walk. Perhaps fatefully, of all the Nobel Laureates who were attending the Meeting, Professor Nash was the one that I most frequently got to meet and talk to in person. We spent that time talking about the places and stuffs in the town, and we soon came across a nice little souvenir shop.

The weather outside was very comfortable, and while we didn't get to look at the thermometer, it should have been somewhere between 15 and 20 degrees (Celsius). Madame Alicia went into the store to browse through the local articles, and Professor Nash and I stayed in front of the store. Professor Nash casually leaned against the tree located next to the postcard stand, and we talked about how the world had changed during the last several decades, and how the transportation and mass transit systems differed from one part of the world to another. After a while, we thought it would be fun to join Madame Alicia in the store, and we entered the store for a good look at the items along with her.

Afterwards, en route to the taxicab station, we were back to the aforementioned building that houses the tourism office and the Foundation. Continued from the previous conversations, I made a comment that the taxi service in Lindau differed from that in the city I live in. Taxicabs in Lindau are stationed in one place and clients can seek them out at the town station, where taxi service providers in Bangkok would seek out potential clients on the street. Professor Nash then told me about how New York taxicabs operated in a similar manner to their counterparts in Bangkok.

Along the way, I had observed how the couple had always been looking out for one another. For the entirety of the time that I had spent with them during and after the Nobel Laureates Meeting, the seemingly minor details contributed to this impression of mine. This might seem rather an implicit observation, but if one had been with them during all of those aforementioned events in Lindau, from the small details you will see how they love, care and genuinely feel concerned for one another during each of the episodes. In my eyes, they represent couples who are best friends and supporters for one another.

After a short while, we reached the station. We secured a cab and, as the Professor and Madame Alicia were getting ready to get back to their accommodation, I felt compelled to perform the "Waii" again as I said goodbye in that afternoon. I will never forget that heart-warming scene where Madame Alicia gave a hearty smile and waved, along with her goodbye, and Professor Nash leaned forward and reciprocated with a kind "Waii", again with a graceful posture coupled with his generous smile and benevolent look in his eyes. Although not all of these said scenes were pictured, properly out of respect for the Professor and his family's privacy, the memory of that day will never be forgotten nor erased from my mind. I will always remember that day of meeting the Professor John Nash and Madame Alicia Nash after the meeting, along with the other great memories in Lindau.

I am of the view that this writing will serve as a piece of information for those who want to know, and/or know more, about the Professor John Nash, especially in scopes apart from his academic talents and as a person of modesty, despite his monumental intellect and genius. Getting to meet him in person, as a young Ph.D. Candidate, I am strongly of the impression that Professor John Forbes Nash Jr., one of the most talented mathematicians and economists in the profession, indeed possessed "a beautiful mind", which was expressed not only through his intellect, but equally in his inspiring, warm and amiable character.

Afterwords

When I first wrote the initial version of this article several years ago, one can tell that it was brimming with excitement, joy and pride, as a re-telling of the wonderful story of the memories we all shared at Lindau 2011, in admiration of the great John Nash, the man of legend and an inspiration of mine. On the other hand, the re-edited version of this article, the one that you are reading, was re-edited in grief to mourn the passing of Professor Nash and his kind wife, his devoted companion throughout his days of joy and misery, Madame Alicia. At first when I heard the news, I could not believe it. I thought it was just an internet celebrity death hoax, origi-

photo by author

nating from some inconsiderate people. I spent several hours checking the validity of the news, and upon learning that the news of their death was real, there was sadness and silence. They were so healthy and energetic, so full of life when I last saw them. I say, to be honest, this is not the ending they deserve. Not at all. I expect that the couple, after all they have been through, deserve many more happy years of companionship and warmth, surrounded by friends and admirers. Such people who have inspired so many and contributed so much deserve a peaceful ending, not this. We know full well that we cannot change what has happened, so I re-edit this article in the hope of sharing the memories of Dr. Nash and Madame Alicia, so as to be a small voice which inspires curiosity and interest about his life and work, which would hopefully encourage the younger generation to learn and build upon his theories, his researches and contributions towards academic and economic progress. There are numerous sources if you'd like to learn more about his life's work, which can be found here:

- For the full presentation of his Lindau lecture on the concept of ideal money, you can find the full video here [http://www.mediatheque.lindau-nobel.org/videos/31344/ideal-money-and-the-motivation-of-savings-and-thrift-2011/laureate-nash-jr]

- A great article about Professor Nash in his remembrance is provided by Charles A. Holt (2015) in his article "John Nash: Flashes of Brilliance in Different Directions", published in the Southern Economic Journal. Here, the article explains how some of his most well-known theories are formulated, based on, for example, the details of bargaining and the need for a prudent selection of assumption, and summarizes the equilibrium concept as the point where players would stay their course even if their opponents' strategies are announced. The contribution of his theories led to the popularity of the well-known "Prisoners' Dilemma" game, where the payoff structure results in the suboptimal equilibrium where both players defect and each receive an inferior payoff compared to what they stand to gain in a successful cooperative outcome.

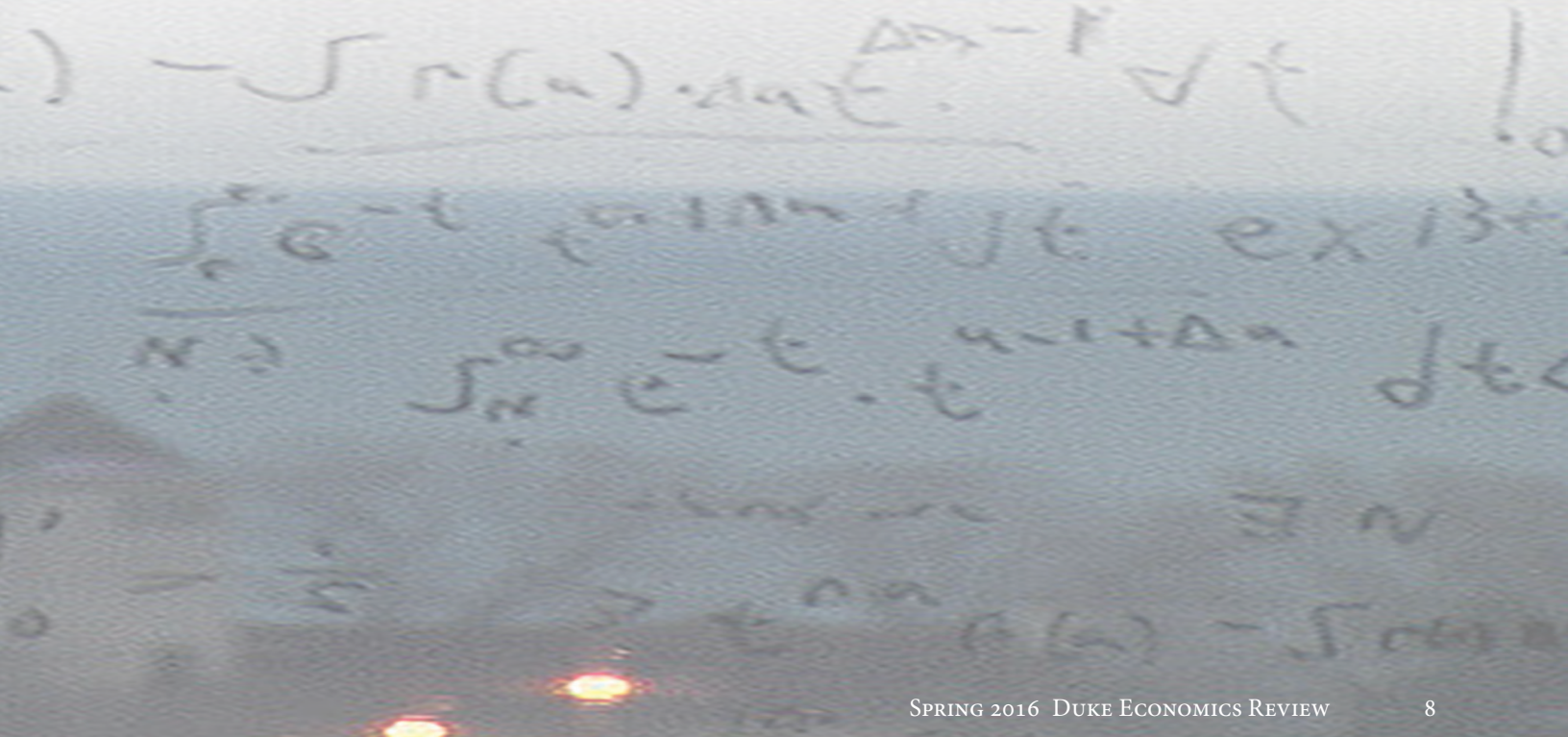
- The concept of "Ideal Money", which leads to the discussion from the plenary lecture early on can be found from Professor Nash's 2002 Work on the same concept.

In my original version of this article, I expressed joy and happiness for them that the couple remained there for one another always. Now I write to mourn their passing and to honor their memories. As fate would have it, while the ending was heartbreaking to all of us, they were together to the very end.

Acknowledgements

I would never consider this article complete and presentable until I give thanks to all the wonderful people who have contributed to it. My heartfelt thanks to Professor Dr. Edward Tower, a mentor, and, if I may, a great friend throughout these years, whose kind suggestions expanded this article from a story in spoken words into an (initially informal) article written for the sake of the good memories, and ultimately into a presentable article you are reading today. Without his advice, all of this would still be in my own memory, and not on this page that you are so graciously taking your time to read as we speak. I thank all my professors at the Faculty of Economics, Chulalongkorn University, who are, similar to the great laureates, my role models that I look up to throughout these many years, and for providing my knowledge and shaping me into what I eventually am today. I thank Ms. Nadine Gärber and all the wonderful people at the Lindau Nobel Foundation Headquarters for making the event very functional and memorable. In addition, I thank the editorial staff at Duke Economics Review for their support in the revising process of this paper.

We young attendants from ASEAN in Lindau 2011 also express our gratitude to the ASEAN Secretariat and the OPEC Fund for International Development for our nomination. We hope that we have fulfilled your expectations of us and did not let you down. My thanks go to ASEAN, OPEC Fund for International Development, and Chulalongkorn University, for my nomination and funding, and for having me there at Lindau 2011.



Behavioral Biases in Mergers and Acquisitions

photo by Vlad Lazarenko

Do Initial Acquirer Bids Anchor Selling Company Boards?

Beau Bressler, Grinnell College

Editing by Aasha Reddy

This paper provides the first systematic empirical evidence for the existence of anchoring effects in the context of actual high-stakes negotiations. I investigate whether a company's board of directors is influenced by behavioral biases when negotiating the sale of their company. Specifically, I investigate whether company boards are psychologically anchored by initial bids made by potential acquiring companies. I find that initial bids, which typically reflect a potential buyer's exploratory bid and thus should not affect the final price paid, are strongly positively correlated with the final sale price. I find that after controlling for a company's fundamental value, selling company boards still tend to be influenced by the value of the initial bid. This supports the anchoring hypothesis, which states that decision makers are disproportionately influenced by an initial piece of information when making numerical estimates.

I. Background and Literature Review

Traditional economic theory assumes that people always act in their own best interests, an assumption referred to as rationality. Do people really behave this way?
A growing body of research suggests that people take mental shortcuts when making decisions. One of these shortcuts is called "anchoring," in which people make numerical estimates relative to an initial numerical anchor (Tversky & Kahneman 1974). Anchoring saves time and cognitive energy, but leads to estimates that are biased away from the true value and towards the anchor. A biased estimate could thus lead someone to act contrary to their best interests. Thus, anchoring is often referenced as an example of irrational behavior.

Anchoring has been observed in a variety of situations such as real estate pricing, art auctions, and university rankings (Northcraft & Neale, 1987; Beggs & Graddy, 2009; Bowman & Bastedo, 2010). Most evidence about anchoring, however, comes from laboratory experiments in which subjects– usually college students– are paid small sums of money to participate (for examples, see Chapman & Johnson, 1999; Galinsky & Mussweiler, 2001; Jacowitz & Kahneman, 1995). Students in these experiments typically lack strong incentive to exert effort so as to make correct estimations. Economic theory predicts that rational actors will balance the benefits of making an accurate estimate against the costs of engaging in more complicated calculations, such as time and mental energy expenditure. Since laboratory subjects have little to gain from making accurate estimates, these experiments do not necessarily reflect irrational behavior. An ideal test of rationality would see if economic actors take mental shortcuts in a high-stakes context. In this paper, I examine one such context: Multi-million dollar corporate takeover negotiations.

The business press contains many anecdotes that support the idea that anchoring occurs in business negotiations. For example, Lovallo, Viguerie, Ulhaner, and Horn (2007) claims that anchoring may cause business negotiators to underreact to new evidence in negotiations. Subramanian (2011) encourages negotiators to make a first-move to anchor the opposing party. Despite these anecdotal claims, scientists still know very little about the extent and prevalence of anchoring in business negotiations. One might expect that business negotiators are trained to avoid being anchored. However, Kaustia, Alho and Puttonen (2008) show that experts estimating stock returns were influenced by anchors. Additionally, Wilson, Houston, Etling, and Brekke (1996) demonstrated in a laboratory setting that even people who were forewarned about anchoring did not show a reduction in anchoring effects. This implies that that we may observe anchoring effects in high-stakes business negotiations even if the negotiators are trained to avoid them.

One of the most important negotiations a businessperson can be involved in is the negotiation over the sale of their company (i.e. "mergers and acquisitions" or "M&A"). M&A negotiations are an attractive laboratory for studying anchoring for two reasons. First, M&A negotiations have high stakes. The average value of a company sale in 2010 was $317 million dollars. So an incorrect estimate by a selling company of their own sale value can be very costly. The stakes are also high from a legal perspective. Corporate boards of directors have a legal duty to their shareholders to maximize their company's value. According to Cornerstone Research, over 90% of proposed mergers in the past five years have been legally challenged by shareholders who claim that their company's board of directors did not do enough to maximize shareholder value. Corporate boards thus have a strong incentive to negotiate aggressively and obtain high sale prices to avoid a lawsuit. Second, relevant data on the negotiation process are available. Selling companies are legally obligated to provide a thorough outline of the pre-announcement negotiation process to the Security and Exchange Commission.

These documents are publicly available, though data is not provided in a research-ready format. By hand-coding information from these documents about the pre-announcement process, I examine anchoring in a novel way.

I investigate whether the initial bid made by a potential buyer in an M&A negotiation psychologically anchors the expectations of selling company boards. To do this, I propose a model of M&A negotiations which accounts for how the selling company board's reservation value can be affected by an anchor. I derive a hypothesis from this model which I test using hand-collected data on the negotiation process drawn from SEC merger.
My main result is that, after controlling for the target's standalone value, the final sale price is positively correlated with the initial bid value.

The remainder of the paper is organized as follows. First, I present a theoretical model of M&A negotiations. Second, I explain how I collected the data I used in my analysis. Third, I discuss the methodological approach and empirical findings. And finally, I draw conclusions from the results and discuss potential limitations of my research.

II. Theory

In this section I propose a model of M&A negotiations that accounts for endogeneity in a selling company board's reservation value. I derive my empirical tests from this theoretical model. To my knowledge, this is a novel approach to modeling M&A negotiations.

First, I treat a selling company's board of directors as a single decision-making unit. This allows me to make simplifying assumptions about the board's reservation value. Second, I assume that the reservation value of the board of selling company i in year t is a function of the selling company's stand-alone value, an M&A markup value, and psychological biases.

Selling company boards seeking to maximize shareholder value will not accept a sale price that is below the company's stand-alone value. However, companies are not sold to simply receive the stand-alone value of the company. Selling companies need to receive a premium to offset the costs of conducting an M&A negotiation and to fulfill their legal duty to shareholders. I include a psychological bias term in light of evidence that value judgments are not retrieved from memory, but constructed in response to information (Chapman and Johnson 1999). This yields the following model of reservation value:

$$RV_i=\beta_2(S_{it-1}+M_t)+\lambda_{it}+\omega_{it} \qquad (1)$$

where RV is the selling company board's reservation value, the lowest price at which they are willing to sell the company, S is the selling company's stand-alone value, M is an M&A markup value, λ is the collection of psychological biases affecting the selling company's board, and ω is an error term that contains other factors that affect the selling company's reservation value. I assume that the stand-alone value of selling company i in year t is equal to the average market value of the company during the year *t-1*. This assumption is founded on the idea, referred to as the Efficient Markets Hypothesis (Fama 1970), that the market price of a company contains all publically available information about a company's value. This assumption accounts for variation in stock prices as well as the effects of market run-ups prior to a sale. I also assume that the M&A markup is equal to the average M&A deal premium in year *t*. A "deal premium" is the percentage difference between the market value of a company and the price that it is sold for. Some companies are sold for a premium below the yearly average, while others are sold for a premium above the yearly average, but I assume that this measurement error is randomly distributed. Therefore, this assumption should not bias the coefficients.

Assuming that an initial bid has anchoring effects on the selling company board, we have that behavioral biases are a function of the initial bid in a negotiation are captured by

$$\lambda_{it}=\beta_1 INITIAL_{it}+u_{it} \qquad (2)$$

where *INITIAL* is the value of the initial bid value for company *i* and *u* is an error term which contains all other psychological biases that affect the reservation value of selling company board *i*. If the initial bid is not an anchor, β_1 would be equal to zero.

The seller's reservation price tells only half of the story of a company sale process. M&A negotiations often consist of long, costly negotiations between the selling company and the potential acquirers to extract additional value from the transaction. These negotiations often result in the acquirer paying more than the selling company's reservation value. To account for this, I model the final sale price as a function of the reservation value. I assume that the final sale price, FP, is equal to the reservation value plus a vector of other characteristics of the negotiation. This yields the following equation:

$$FP_{it}=RV_{it}+\alpha_{it}+\varepsilon_{it} \qquad (3)$$

where α represents negotiation characteristics that affect the final price, and ε is an error term. Inserting Equation 2 into Equation 1, we have that

$$RV_{it}=\beta_2(S_{it}+M_t)+\beta_1 INITIAL_{it}+\omega_{it}+u_{it} \qquad (4)$$

Finally, inserting Equation 4 into Equation 3, we have that

$$FP_{it}=\beta_1 INITIAL_{it}+\alpha_{it}+\varepsilon_{it} \qquad (5)$$

where

$$\varepsilon_i=\beta_2(S_{it}+M_{ai})+\varepsilon_i+u_{it} \qquad (6)$$

This model implies that the initial bid value will affect the final sale price.

Inserting Equation 6 into Equation 5 and grouping terms yields the basic estimating equation:

$$FP_{it}=\beta_2(S_{it-1}+M_{ai})+\beta_1 INITIAL_{ai} + \alpha_{it}+\varepsilon_i+u_i \qquad (7)$$

Notice that $dFP/(d\beta_1)>0$ implies that an increase in the initial bid value leads to an increase in the final sale price. My empirical analysis therefore seeks to estimate the value of the β_1 coefficient.

III. Data and Methodology

A. Data

All variables are constructed from data

from SEC EDGAR, Bloomberg, and the Compustat databases. I collected data on bids, sale prices, and other pre-announcement negotiation variables from proxy statements submitted by companies to the SEC after the announcement of a company sale. My sample is comprised of 33 randomly selected company sales with the following characteristics:

- The selling company is a publicly-listed company in the S&P 1500.
- The total value of the sale is greater than $1 million.
- The sale occurred during 2010.
- The initial bid is provided in "per-share" terms within the SEC documentation.

I collect the dependent variable and independent variable of interest, as well as control variables, from the 'background of the deal' section of SEC proxy statements. These documents provide a thorough summary of the pre-announcement negotiation process, including all bids and indications of interest. These 'background of the deal' documents often provide information about other negotiations that the selling company was previously engaged in within the last few years. It takes about an hour to read each document and code it into a format that is usable by a statistical package. It is thus not surprising that prior research on anchoring has not used this data. In my analysis, I focus on the most recent "bidding round" before sale. I define the end of a bidding round as one of the following events:

- All potential buyers cease negotiations with the selling company.
- The selling company terminates the sale process altogether.
- The selling ceases negotiations with each potential buyer.

I define the "initial bid" variable as the first observed bid in a bidding round. In a typical M&A negotiation, bidders often offer verbal or written non-binding indications of interest before eventually making written bids (Hansen 2001). I assume that these two forms of bidding are indistinguishable. This assumption is justified for two reasons. First, selling company boards tend to respond to indications of interest in the same way that they respond to bids. If the valuation in the indication of interest is too low, board will urge the bidder to increase their valuation, present a counter-offer, or remove the bidder from the negotiation entirely. If the valuation is satisfactory, the board will occasionally ask to proceed at the indicated price. The same is true for formals bids. Second, research on anchoring suggests that even if the valuation is deemed an "indication of interest," it will still have anchoring effects. The selective accessibility model proposed by Jacowitz and Kahneman (1995) suggests that even an uninformative anchor will affect the negotiator's eventual valuation.

Indications of interest are often reported in proxy statements as a range of values. It is reasonable to assume that the selling company board members will care about how much the bidder might be willing to pay rather than how little they might pay. I therefore record the initial bid value as the top of the range if the initial bid is reported in a proxy statement as a range.

B. Empirical Model

In this section I test the hypothesis that higher initial bid values contribute to higher sale prices. Recall equation (7):

$$FP_{it} = \beta_2(S_{it-1} + M_{ai}) + \beta_1 INITIAL_{ai} + \alpha_{it} + \varepsilon_i + u_i$$

Ideally, I would simply estimate the value of β_1 in this equation. However, to ensure a casual effect, I control for variables contained within the error term, ε_i, that may be correlated with the dependent variable, *FP*, and the independent variable of interest, *INITIAL*.

I therefore estimate the following equation, which I derived in section II., using Ordinary Least Squares:

$$FP_{it} = \beta_0 + \beta_1 INITIAL_{it} + \beta_2 Valuation_{it} + \beta_3 TSIZE_{it-1} + \beta Numbidders_{it} + \beta_5 Unsolicited_{it} + \beta_6 Lnbeta_{it} + \mu_m + \varepsilon_{it}$$

where:

FP_{it} = The per-share value of the final sale price of selling company *i* in year *t*

$INITIAL_{it}$ = The per-share value of the initial bid for company *i* in year *t*

$Valuation_{it} = (S_{it-1} + M_{ai})$, a proxy for the expected sale value of company *i* in year *t*

$TSIZE_{it}$ = The natural log of the average market capitalization of company *i* in year *t-1*

$Numbidders_{it}$ = The number of bidders in the final bidding round for company *i* in year *t*

$Unsolicited_{it}$ = A dummy variable indicating whether the initial bid for company *i* was unsolicited

$Lnbeta_{it}$ = The natural log of the average beta coefficient of company *i* in year *t*

μ_m = Industry fixed effects

$\varepsilon_i = (\varepsilon_i + u_i)$, a combination of two stochastic error terms

A positive value of β_1 would indicate that the size of the initial bid is associated with a higher final sale price, as predicted by the anchoring hypothesis. The FP equation contains a variety of control variables that ensure that the association between initial bids and final price is indeed casual. Each of the control variables included is potentially correlated with both the size of the initial bid and the final sale price. For example, we would expect that the *Valuation* variable to be positively correlated with both the initial bid price and the final sale price. This variable serves as a proxy for the actual value that the selling company board would accept in the absence of psychological biases. Thus, a large valuation implies an inherently more valuable selling company, which should illicit larger bids as well as a higher final price. Omitting this variable would therefore positively bias the β_1 coefficient.

Each of the control variables included in the multivariate regression is included to avoid potential omitted variable bias. For example, one potential concern is that boards of directors of differently sized companies may behave differently. All else equal, a larger company likely has more outside options than a smaller company. This gives the selling company board higher negotiating power, and should therefore illicit higher initial bids. For a similar reason, larger selling companies with higher negotiating power might be able to extract higher final bids as well. Thus, we include the $TSIZE_{it-1}$ variable to capture the potential effect of company size on our initial and

final bid values. Omitting the selling company market capitalization would bias the β_1 coefficient upwards.

Another potential concern would be that the number of bidders in a negotiation will affect the ability of the selling company board to negotiate a high final sale price. On one hand, if a potential bidder offers a high initial bid, the selling company board may be less likely to bring in more bidders. On the other hand, one might expect that more competition in the bidding process might drive up the final sale price. Thus, *Numbidders* would likely be negatively correlated with the initial bid value and positively correlated with the final sale price. Since the signs of these correlations are different, omitting the number of bidders from the model should negatively bias β_1.

IV. Main Findings and Robustness Checks

Table 2 reports the results of the initial regression. Column (1) reports the results of the bivariate regression between the initial bid and the final sale price. Column (2) reports the results of the same regression when we include the expected valuation of the target company. Column (3) reports the results of the multivariate regression, excluding controls for industry. Column (4) reports the complete multivariate regression results. In each model, the estimated coefficient on the initial bid value is positive and significant at the .001 level. The estimated coefficient of the $INITIAL_{it}$ variable in the complete model implies that a 1 dollar increase in the initial bid value leads to a 1.18 dollar increase in the final sale price. Otherwise, only expected sale price, the beta variable, and one of the industry dummies were statistically significant. These results are consistent with the hypothesis that the initial bid anchors the selling company board (see table II).

One interesting finding that emerged from the regression is that the selling company valuation variable, in every model in which it is included, is statistically significant, but with a negative coefficient. This implies that the Valuation variable may not capture the true company M&A value in the sample. To correct for this potential specification error, I alter the model specification by dividing the estimation equation by the $Valuation_{it}$ variable. Thus, we have the following alternative estimating equation:

$$FP_{it}/Valuation_{it} = (\beta_0+\beta_2) + \beta_1(INITIAL_{it}/Valuation_{it}) + \beta_3(TSIZE_{it-1}/Valuation_{it}) + \beta_4(Numbidders_{it}/Valuation_{it}) + \beta_5(Unsolicited_{it}/Valuation_{it}) + \beta_6(Lnbeta_{it}/Valuation_{it}) + \mu_m + \varepsilon_{it}$$

In this alternative regression, β_1 represents how a change in the initial bid premium affects the final "deal premium" or sale premium, where a premium is defined as the value-per-share of the bid divided by the estimated value-per-share of the company on the M&A market.

See table III. Column (1) presents the bivariate regression, while column (2) presents a multivariate regression that includes all controls except for industry controls. Column (3) presents the complete model. In each model, the initial bid premium has significant, positively correlated effect on the final sale premium. In the complete model, the estimated coefficient of interest implies that a 1% increase in the initial bid premium leads to a 1.31% increase in the final sale premium. This supports the anchoring hypothesis. Thus, both specifications of the estimating equation yielded an estimated β_1 value that is consistent with an anchoring hypothesis.

IV. Conclusion

This paper used hand-collected data to show that economic actors in high-stakes business negotiations may not be perfectly rational. My main finding demonstrates that even in cases in which people have every incentive to behave rationally, they are unable to do so. This aligns with the theory of bounded rationality, the theory that people try to act in their best interest but are limited by imperfect information and inadequate cognition. Selling company boards have a lot to lose by making inaccurate estimations of their value, but their valuations are still biased by an anchor.

VI. Tables

Table I. Sample Characteristics

This table reports mean characteristics for the deals contained in the sample. Data are hand-collected from SEC proxy statements and Bloomberg. Valuation is equal to the average company stock price in the previous year plus the average merger premium in the given year. Final sale premium is equal to the final sale price divided by the company valuation, while initial bid premium is equal to the initial bid value divided by the company valuation.

Table II. Effect of Initial Bid on Final Sale Price

This table reports regression results of the initial bid for a company on the final sale price of that company. Column (1) reports the bivariate regression results. Column (2) controls for the estimated value of the selling company, calculated as the to the average company stock price in the previous year plus the average merger premium in the given year. Column (3) contains the full multivariate regression results, excluding industry fixed effects. Column (4) contains the multivariate regression results after controlling for industry fixed effects.

Table III. Effect of Initial Bid Premium on Final Sale Premium

This table reports regression results of the initial bid premium for a company on the final deal premium of that company. Final sale premium is equal to the final sale price divided by the company valuation, while initial bid premium is equal to the initial bid value divided by the company valuation. Column (1) reports the bivariate regression results. Column (2) contains the full multivariate regression results, excluding industry fixed effects. Column (3) contains the multivariate regression results after controlling for industry fixed effects.

Table I. Sample Characteristics

Variable	Mean	Std. Dev.	Min	Max
Sale Price	21.32	28.64	0.41	150
Initial Bid	19.53	26.70	0.34	140
Valuation	12.80	16.58	0.73	83.88
Final Sale Premium	0.50	0.34	0.02	1.33
Initial Bid Premium	0.39	0.39	-0.71	1.65
Log of Market Capitalization	5.14	1.79	1.76	8.54
Unsolicited Bid	0.30	0.47	0	1
Log of Beta	0.02	0.63	-1.55	0.87
Number of bidders	2.28	1.76	1	8

Table II. Effect of Initial Bid on Final Sale Price

	Sale Price			
	(1)	(2)	(3)	(4)
Initial Bid	1.071***	1.173***	1.169***	1.179***
	(86.71)	(32.19)	(32.00)	(34.31)
Value of Seller		-0.173**	-0.171**	-0.175**
		(-2.95)	(-2.97)	(-3.30)
Log of Market Cap			0.128	0.172
			(0.72)	(0.84)
Unsolicited Bid			-0.986	-0.989
			(-1.41)	(-1.51)
Log of Beta			-0.906	-1.164*
			(-1.76)	(-2.33)
Number of Bidders			0.183	0.116
			(0.99)	(0.66)
N	33	31	31	31
R^2	0.996	0.997	0.998	0.998

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table III. Effect of Initial Bid Premium on Final Sale Premium

	Final Sale Premium		
	(1)	(2)	(3)
Initial Bid Premium	1.275***	1.280***	1.313***
	(25.24)	(22.68)	(25.30)
Log of Beta Value		-0.0611	-0.128*
		(-1.18)	(-2.51)
Log of Market Cap		-0.0138	-0.0136
		(-0.35)	(-0.37)
Unsolicited Bid		-0.197	-0.181
		(-1.66)	(-1.72)
Number of Bidders		0.190	0.158
		(1.94)	(1.84)
N	33	31	31
R^2	0.954	0.964	0.976

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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Should the Equities in the North Carolina State Employees' Pension Fund be Indexed or Actively Managed?

> "In Investing, You Get What You Don't Pay For."
>
> — John C. Bogle, founding CEO of the Vanguard Company. This is the title of his talk to the World Money Show 2005.

> "Money management is probably what is generously called a zero sum game, which is to say, zero before management fees and transaction costs."
>
> — Jeremy Grantham (2006, p.3).

Professor Edward Tower, Duke University

Editing by
Kevin Bao and Caroline Lai

Ron Elmer is running for Treasurer of the State of North Carolina. He is aware of Jack Bogle's and Jeremy Grantham's quotes that introduced this paper, which make the point that the high expenses and high turnover associated with active management of portfolios reduce returns. Part of Elmer's platform is to replace active management of stock mutual funds, private equity, real estate funds, real estate partnerships, hedge funds and commodity funds with an index investment strategy, managed in house. He tells me that he doesn't believe real estate even belongs in the portfolio as a separate asset class. The S&P 500 and Wilshire 5000 include REITs and the current bond portfolio of the Pension Fund includes mortgaged based securities.

In this article I estimate what the gain would be, if any, from a strategy which is close to Mr. Elmer's proposal. I ask what the returns would have been if the state had simply invested in Vanguard index funds. Mr. Elmer tells me that he believes that he can do even better than this Vanguard index strategy by managing the stock portfolio in house, so perhaps I have underestimated the gain from his strategy. Should he be elected and manage the stock portfolio in house, I suggest he use a benchmark consisting of Vanguard fund returns. Other pension funds might find comparison with the same benchmark to be useful as well.

Mr. Elmer's phone, email, and campaign web site are:
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info@ElmerForTreasurer.com
www.ElmerForTreasurer.com

2. What does this paper do?

North Carolina Department of State Treasurer (2009-2015) reports the returns of the various components of the State of North Carolina Employees' Pension Plan through the second quarter of 2015 as well as year by year returns. These are reproduced in the appendix to this paper. "The NC Retirement Systems Division administers four major retirement systems and several smaller systems and supplemental funds" (State Treasurer, 2015). I work with this collection of retirement funds.

Vanguard is a large company famous for providing low-cost equity and bond index mutual funds. So a strategy worth investigating is the effect of replacing the current management of the NC State Employees' Pension Fund with low-cost Vanguard index funds.

3. What Do The Calculations Show?

Exhibits 1-11 and A3 show the effects of replacing components of the NC State Employees' Pension Fund with index funds from Vanguard. Here is how to read them.

Exhibits A1 and A2 report the data I worked with. They present nominal returns. Large and stable real returns are what pension funds should be concerned with, so all the other exhibits report real returns. "Real" returns are the returns after nominal returns are adjusted for inflation.

The returns of assets that have relatively high returns, low standard deviations of return, and low cumulative falls in value are bolded.

Exhibit 1 shows the impact of replacing the pension plan's "inflation protection" with Vanguard's Inflation-Protected Securities, Admiral Share Class, with Nasdaq ticker VAIPX. The Vanguard fund has higher av-

erage returns for each of the 7 time periods ending in mid-2015, a lower standard deviation of real return and a lower maximum cumulative decline in real value.

Throughout the paper I use the Admiral share class of the Vanguard funds. Admiral shares have lower expenses than the corresponding Investment share class, and the hugeness of the NC portfolio means that NC would likely qualify for the lower expenses. The Admiral shares were introduced more recently than the more expensive Investor share classes. When the Admiral share class did not exist for the entire time period, for the pre-Admiral share class period, I used the return for the Investor share class, and increased that return by the 2015 expense ratio of the investor share class minus the 2015 expense ratio of the Admiral share class. This approximates what the return of the Admiral share class would have been if it had existed for the entire time span. The expense ratio of Vanguard's Institutional share class is typically 0.1 % less than that of the Admiral class, and I expect that the NC Pension Fund would qualify for this lower expense ratio. One contributor to the high returns of Vanguard funds is that Vanguard lends out its securities to short sellers and charges for this service. Ron Elmer tells me that the NC State Pension fund does the same, so lending does not account for any of the excess Vanguard returns.

Exhibit 2 illustrates that the investor in inflation protection would have had to invest $1.52 real dollars in mid-2009 to cumulate to one real dollar in mid-2015, whereas the investor in VAIPX would have had to invest only $0.87 real dollars to cumulate to one real dollar in mid-2015. Thus VAIPX has the higher six-year return. This is illustrated in Exhibit 3, which presents the geometric average annual real rate of return for various start dates. This is the return that if prevailed every year until mid-2015 would have yielded the observed cumulative return.

The series start in different years, but all end in mid-2015. To compare recent performance, I show how many real dollars cumulate to one real dollar in mid-2015, rather than how many dollars one dollar at the start cumulates to in mid-2015.

In all of the graphs, the Vanguard values and returns are indicated by the squares and lines in Vanguard's color: crimson.
Exhibits 4, 5, and 6 provide the same analysis for Core Real Estate versus VGSLX, the Vanguard REIT Index Fund, Admiral share class. Over the most recent four years, the geometric average returns of the two are a virtual tie, but over longer periods VGSLX wins. VGSLX has a higher standard deviation of real annual return and a 9.5 percentage point higher maximum cumulative drop in real value than Core Real Estate, but the larger return of VGSLX means that its maximum cumulative three-year drop in real value is less than that of Core Real Estate by 5.1 percentage points. Vanguard only established a REIT in May 1996, so the comparison of the Core Real Estate versus VGSLX dates back only to mid-1996. Mr. Elmer suggests replacing the real estate investments with a broad-based equity investment that includes REITS, rather than investing in a REIT fund. VTSAX outperformed VGSLX over the last five and ten years, so using VGSLX instead of VTSAX as the alternative to Core Real Estate biases my five and ten year calculations of gains from indexing downward. For the most recent 19-year period, Core Real Estate's average return of 4.425%/year is exceeded again by both VGSLX's 7.96% and VTSAX's return of 5.98%, but here VGSLX (the Vanguard REIT) is the best performer.

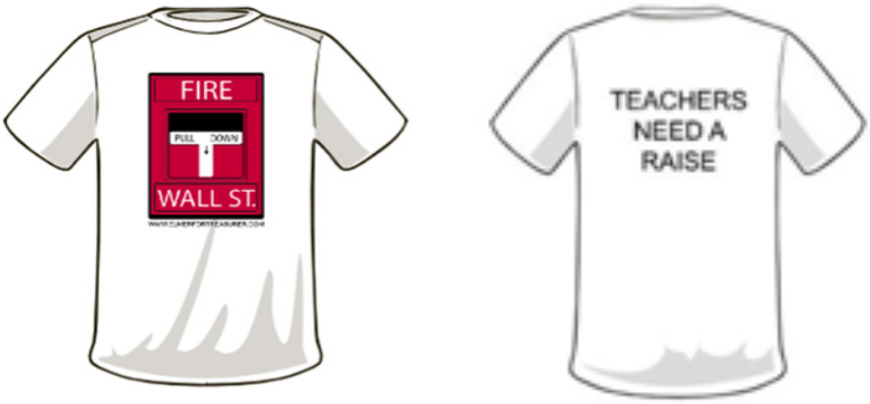
Exhibits 7, 8, and 9 provide the same analysis for Public Equity and Private Equity versus VTSAX, Vanguard Total Stock Market Index, Admiral Shares. This Vanguard fund invests in the broad US stock market. Its benchmark is the CRSP US Total Market Index. VTSAX outperforms public equity for each of the time spans to mid-2015. It also outperforms private equity for each of the time spans except mid-2014 to mid-2015. Its maximum cumulative fall in real value and its standard deviation of real return are only a hair larger than those of public equity. It fares worse on these two measures than does private equity but private equity is not valued by the market, so its measured low risk is suspect as discussed in section 6.

Exhibit 10 records the geometric average real returns for the five asset categories over the longest time span for which we have annual returns, and for the same periods, the returns of the corresponding Vanguard funds. The Vanguard return minus the pension fund return is multiplied by the assets in each category at mid-2015 to arrive at the saving from indexation.
I did not analyze Non-Core Real Estate in Exhibits 1 through 9. However, data from the NC Department of State Treasurer reports geometric average nominal returns on Core and Non-Core Real Estate over the 10 years through June 2015 of 5.6%/year and 5.2%/year respectively, so Non-Core underperformed by 0.4%/year. Over the longest period Core Real Estate's real return was 4.42%/year, so for that same period we assume in Exhibit 10's calculation that Non-Core Real Estate's real return was 4.02%/year.

The Inflation Sensitive and Diversifiers category returned 2%/year nominal over the 10 years prior to mid-2015. Over the same period, VAIPX returned 3.63 %/year. After adjusting for inflation, the figures are 0.523%/year and 1.534% per year respectively. I used these figures in Exhibit 10, rather than the figures of Exhibit 1, which produced a difference in favor of indexation of 9.12%/year. This choice makes indexation appear less appealing, but still better than the Inflation Sensitive category. However, the recent abysmal performance of the Inflation Sensitive category should not be ignored.

For the Private Equity, the figures of Exhibit 7 produced a difference in favor of indexation of 5.21% /year. We chose to use the 15-year figure of Appendix A2, again ending in mid-2015, which produced a difference in favor of indexation of 4.15%/year. Again, this choice makes indexation appear less appealing but still better.

I did not analyze the impact of replacing hedge funds with a stock index, becauseI do not have the 2014-2015 report on their return and I do not know what proportion of the pension portfolio they made up at any time. The annual returns of hedge funds in the pension portfolio are listed in Exhibit A2 only for the six years prior to the fiscal year ending in mid-2015. Moreover, there is no report on their return for the fiscal year ending in mid-2015. It may be that the pension fund no longer invests in them. If so, that may be a good thing as their six year geometric average annual real rate of return ending in mid-2014 was -0.778%/year, while VTSAX's six year geometric average annual real rate of return over the same period was 17.616 %/year.



Ron Elmer's Campaign Tee Shirt illustrates his proposal to index the NC State Employees' Pension Fund and Use the Savings for a Worthy Cause.

4. Do the Pension Mangers Make Prescient Asset Reallocations?

Does the pension plan increase the share of its funds in asset classes just before they have high returns relative to other assets? To figure this out, I draw on Exhibit A2's data from mid-1996 through mid-2015. I regress the annual continuously compounded nominal return of the pension fund on the annual continuously compounded nominal returns of the three components of the plan for which we have annual returns: income, equity, and real estate plus a constant term. The returns are expressed as %/year. I use continuously compounded returns for two reasons. First, I assume that the pension plan is rebalanced frequently throughout the year, so, for example, if equities perform well relative to other assets, the pension manager sells equities or invests new funds in bonds to bring the share of the portfolio in equities throughout the year back to its initial level. Second, the geometric average continuously compounded return equals the average continuously compounded return, but that is not true for annualized returns. Thus the average continuously compounded return is meaningful, whereas the average annualized return is not.

We know that the shares of the assets in the pension fund must add up to one, so I constrain the sum of the coefficients on income, equity, and real estate to add to one. The coefficients represent average portfolio shares in the three asset classes. The regression minimizes the sum of the squares of the prediction errors. If the constant term is positive, then we can conclude that the managers increase shares of assets just before they perform relatively well. We use Microsoft solver to perform our constrained regression. Our regression equation is

Pension Return =
0.438*Income Return + 0.483*Equity Return + 0.079*Real Estate Return – 0.312.

This regression tells us that on average the pension fund holds 44% of its assets in bonds (the income return), 48% in equity, and 8% in real estate. These are quite close to the current allocation from Exhibit 10. The regression also tells that the return of the pension fund is 0.312%/ year less than it would be if the asset shares were constant. Thus our point estimate is that managers adjust their asset holdings in the wrong direction, shrinking the allocation to asset classes just before they appreciate, and this drags down the portfolio return by 0.312% per year. The R for the regression, which is the correlation between the actual portfolio returns and the calculated returns is 0.971 (a measure of goodness of fit of the regression). The R square is 0.942.

Another interpretation of the negative constant term is that the other components of the pension fund produce inferior returns to those we put into the regression equation. These other components are the rightmost 9 columns of Exhibit A2, including private equity and hedge funds.

Microsoft Excel's "t-test: paired two sample for means" test tells that the true value of the constant term is negative with a probability of 75.1% and positive with a probability of 24.9%. Thus the coefficient is not significant at conventional levels, so if the NC Pension strategy stays the same in the future, my best guess is that the future constant term will be negative with that same probability of 75.1%. The lesson of this calculation is that there is no evidence that managers make prescient asset reallocations.

5. Andrew Silton and I share frustration that the NC pension fund doesn't provide more orderly data.

It is frustrating that the investment categories in Exhibits A1 and A2 are not identical, and that the Pension Fund does not provide annual returns in Exhibit A2 that correspond to the long periods in Exhibit A1. Stilton is the former Chief Investment Officer of the North Carolina Retirement System. In a June 12, 2015 News and Observer article titled "Reform needed to get clearer view of NC pension fund." he explains his frustration. Here are some excerpts from his article:

"For about a month, I've been preparing to write a column based on the detailed performance and fee schedule released by State Treasurer Janet Cowell's office for the North Carolina pension plans. As I've previously written, the total fees came to just under $500 million, an 18 percent increase. Like almost every public pension plan in the country, the increases have been driven by the shift to alternative investments. That's really not news, and the performance and fee data aren't timely, since they cover a period that ended on June 30, 2014. Regrettably, it takes the Treasurer's Office more than 10 months to release information that is available internally within 30 days.

I discovered that the treasurer has constructed any number of windows into the performance, risks and allocations of the pension plan, but the panes are made of frosted glass, distorting the information.

For starters, North Carolina uses a mélange of asset classes and categories. … As pension plans and individual investors have moved into alternative investments, the proper use of asset classes has been distort-

ed by mixing in strategies that hide the true exposures of the investment portfolio.

The good news is that the asset allocation looks like a modestly conservative public pension plan. The bad news is that it is heading deeper and deeper into the world of hedge funds and private equity."

6. Andrew Silton points out that the reported riskiness of private equity is wrong.

"Alternative investments are supposed to represent investment nirvana. Money managers have been promising investors that a mix of private equity and hedge funds will boost returns and reduce risk at the same time. The Cowell's (sic) own risk report shows how badly she is being misled. The pension's risk report for the three-year period that ended in March states that conventional equities have fluctuated by an average of 10.5 percent, while the private equity has varied by only 3 percent over the same period. This relationship, known as standard deviation, is completely wrong.

Private equity, given its leverage and illiquidity, is 30 percent to 50 percent riskier than public stocks, which is why investors expect higher returns from the asset class. It's no small wonder that the state treasurer is moving more and more money into alternatives. Her risk reports are telling her that opportunistic real estate, credit and inflation have a risk of 3.2 percent, 2.8 percent and 5.7 percent respectively. I'd be tempted to invest in these asset classes and strategies if the risk was only a fraction of plain old stocks.

But that's not the full story. The reason that the risk statistics look so attractive is that the treasurer's consultants and staff are mixing market and accounting data. Actual market movements are being used to measure stock and bond risk, while estimates taken from accounting statements form the basis for measuring the risk of most other investments. Whether you are the fiduciary for one of America's largest pension plans or a small retail investor, there's a good rule to invest by. If a particular investment offers a big reward relative to its risk, the data is distorted and/or the risk isn't being captured by the risk statistic."

7. Would Indexing Make Much Difference to North Carolinians?

Exhibit 10's estimation of the gain from indexing is $781 million per year using the mid-2015 asset figures. More precisely, we should refer to the strategy as Vanguardizing since VAIPX and VGSLX are not index funds, but they share the low expenses, wide diversification and low transactions costs of index funds. However, VTSAX does index the entire stock market.

In order to put this into perspective, we draw on Exhibit 11. This figure translates into a saving of 3.59% of the annual NC State Budget or $82 per capita saving for each North Carolina Resident. Should the savings be allocated to raising teachers' salaries, the saving would amount to $8,031 per teacher per year, which would be an 18.14 % salary increase.

8. What would the gain from indexing be if the future looks like the recent past?

Exhibit A3 asks what the gain from indexing would be if the return differential in the future is the same as in the recent past. That Exhibit records the same calculation procedure as in Exhibit 10, except the returns are those for more recent periods. Over the most recent five years ending in mid-2015 the return differential times mid-2015 assets is $2.492 billion per year, and over the most recent ten years ending in mid-2015 it is $969 million per year. I was initially reluctant to report these numbers because they seem too good to be true. They may reflect the fact that some authors have found that over time index funds have increased their margin of performance over that of managed funds. See, for example, Tower (2009). In any case, they indicate the gains that indexing would have offered in the recent past.

9. NC State's Professor Richard Warr Reaches a Similar Conclusion With a Different Approach

Warr(2016) by looking at management fees, incentive fees and trading costs reaches the conclusion that "the NC Pension Fund could save $500 to $900 Million annually." These figures are in the same range as my estimates of $781 million, $969 million, and $2.492 billion annually.

10. Reckoning with the Deadweight Cost of Tax Collection

Taxes inflict a deadweight cost on the economy, dragging down output. A reasonable measure of the deadweight cost of taxation in the US (the marginal welfare cost of taxation) is that for every extra dollar of taxes collected by the sorts of taxes that are likely to be imposed, there is a deadweight loss of 35%. A selection of estimates of the marginal welfare cost of taxes is presented in the references. Stuart (1984) estimates that for the United States it is between 20.7% and 24.4%% of the tax collected. Ballard et al. (1985) estimate it to be between 17% and 56% percent. Judd (1987) finds very high MWC's for the taxation of capital, and Gilbert et al. (2011) show how the MWC depends on how the taxes are levied intertemporally.

Thus, should the pension plan saving be used to shrink the tax schedule, households will gain more than the cut according to the schedule. These are called "dynamic" gains. Moreover, the increase in NC GDP will cause NC tax revenues to rise, partially offsetting the initial tax cut, which means that the reduction in the tax schedule should exceed the pension savings. Even if taxes are not lowered by this proposed reform, the proposed reform may enable the slowing of tax increases.

Similarly, the dynamic gains from increases in productive government expenditure on public goods mean that the benefits to North Carolinians are greater than the extra spending.

11. Cumulative Gains

I have presented the savings over one year from indexation. But these are recurrent gains, not one time gains. Suppose indexation allows an individual just starting her work life to experience a 1 real dollar tax reduction that remains in force every year. If she enters the workforce at age 25 and retires at age 65 how much money will that dollar build to over those 40 years? For real rates of return continuously compounded of 0%/year, 2%/year, 3%/year and 4%/ year, the tax saving cumulates to $40, $49, $61, $77, and $98 real dollars at retirement. Historically the stock market has returned 6% real per year, sometimes referred to as Siegel's constant, after the Wharton Profes-

sor Jeremy Siegel. However, given today's lower real interest rates and high stock market valuation, we can expect lower returns today. A 4% real return from the stock market and a 1% real return from the bond market are perhaps reasonable guesses. Thus the family who saves $82 real dollars in taxes per year, if it invests in stocks in a tax-sheltered account could expect to have $8,118 extra real dollars at retirement. If it invests in bonds in a tax-sheltered account it could expect $4018 extra real dollars at retirement. Should the savings be realized in higher teachers' salaries, the retirement assets would be calculated in the same way. For example, in the unlikely event that all the saving were passed on to teachers, the $8,031 real dollars if invested in the stock market each year, would cumulate to $794,069 real dollars at retirement, and if invested in bonds, would cumulate to $393,519.

12. Conclusion

These calculations are rough, in part because of lack of better data. However, they, along with the logic of Andrew Silton, do suggest that Mr. Elmer's proposal for indexation of the Equity part of the portfolio of North Carolina is worth serious consideration. It would be useful to perform similar calculations for other states too.

13. Summary

Our various calculations (from Exhibits 10 and A3) indicate indexing the equity part of the NC Pension fund would have increased returns by approximately $781 million per year (in the 3 years ending mid-2015), $2.492 billion per year (last 5 years to mid-2015), and $969 million per year (last 10 years to mid-2015). This is an increase in the return by between 0.87%/year and 2.78%/year.

The most conservative of these estimates translates into a saving of 3.59% of the annual NC State Budget or $82 per capita saving for each North Carolina Resident. Should the savings be allocated to raising teachers' salaries, the saving would amount to $8,031 per teacher per year, which would be a 18.14 % salary increase. These are annual figures. The gain of $781 million per year translates into $7.810 billion over a decade. The likely growth of the pension plan makes this figure a slight underestimate.

14. Postscript: Reply to Comments

Ron Elmer did not win the Democratic primary. Elmer's share of the vote in the Democratic Primary for Treasurer was 41.5% to Dan Blue III's 58.5%.

Since I posted this paper as a working paper, it has stimulated a column by Mel Lindauer in *Forbes* (http://www.forbes.com/sites/theboglehead sview/2016/03/15/indexing-state-pension-funds/#2bd0c546399a), an article by Tim Storrock in *Fundfire* and an article by Dan Solin in the *Huffington Post* (http://www.huffingtonpost.com/dan-solin/a-plan-to-stop-the-pensio_b_9398016.html). Tim Sorrock wisely sought comment by the NC State Treasurer's office on the paper. Kevin SigRist of North Carolina's Department of State Treasurer office made two crucial comments on the paper. The public equities consist of both US and foreign equities, and in recent years the foreign stock market has had lower returns than the US market. Thus, a benchmark that better captured the Fund's style would be a mix of US and foreign stocks.

He also tells me that the inflation protection category includes energy stocks and commodities, so using the Vanguard Inflation Protected Treasury fund as the benchmark for that category does not capture the style of assets that the pension fund invested in.

I had wondered how it was that the "investment grade fixed income" category generated such high returns. He tells me that "High returns in the investment grade fixed income asset class have been due to two factors:

1. It has a longer duration (~8 years) which makes it particularly sensitive to falling/rising market interest rates. The long duration, compared to broad indices like the Barclays Aggregate Bond Index, is largely a result of our asset liability analysis and ability to ride-out shorter-term mark-to-market volatility.

2. The asset class maintains an allocation of around 30% U.S. Government issued bonds. During "risk-off" environments, the market flight to quality causes outsized returns. This portion of the asset class is also viewed as an important hedge against the risk of deflation."

What should readers take away from the interchange? To some extent this paper compares apples and oranges. The study shows that several of the simple indexing strategies used in this study out-returned the roughly comparable asset categories in the pension fund. Other indexing strategies would have under-returned. I hope to extend this paper to explore with better data whether indexing strategies with the same risk profile would have out-returned the State of North Carolina Employees' Pension Plan. I am grateful to the Treasurer's office for promising me additional data. I think it would be helpful for these data to be regularly posted on the NC State Treasury's web site, and that it is desirable to encourage all state pension funds to provide at least the same level of transparency that is required of mutual funds.

Exhibit 1. Inflation Protection vs VAIPX. VAIPX Returns higher with lower standard deviation of real return.

	Cumulative Real Value		Geometric Avg Return %/yr	
Year	Inflation Protection	VAIPX	Inflation Protection	VAIPX
2015	1	1		
2014	1.09	**1.02**	-8.11	**-1.94**
2013	1.03	**1.00**	-1.30	**0.19**
2012	1.08	**1.07**	-2.42	**-2.17**
2011	1.22	**0.97**	-4.95	**0.76**
2010	1.52	**0.94**	-7.99	**1.32**
2009	1.52	**0.86**	-6.74	**2.48**
Std deviation of annual real return %			8.12	**5.76**
Largest cumulative decline in value %			34.22	**6.72**

Exhibit 2. Inflation Protection vs VAIPX. Cumulative Real Value. Fraction of Mid 2015 Value. Low means high subsequent return.

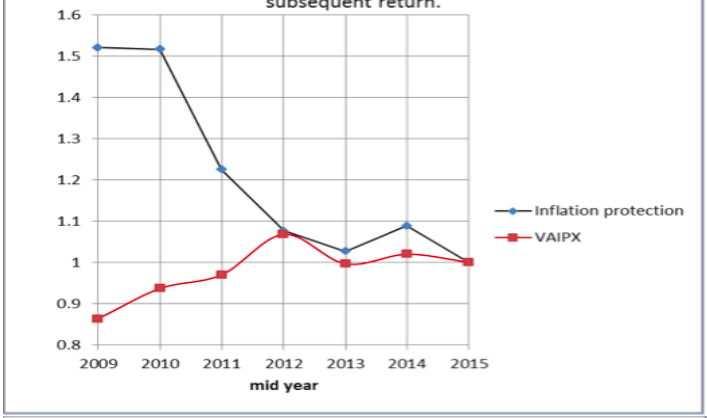


Exhibit 3. Inflation Protection VAIPX Geometric Average Annual Real Rate of Return through Mid 2015. %/yr.

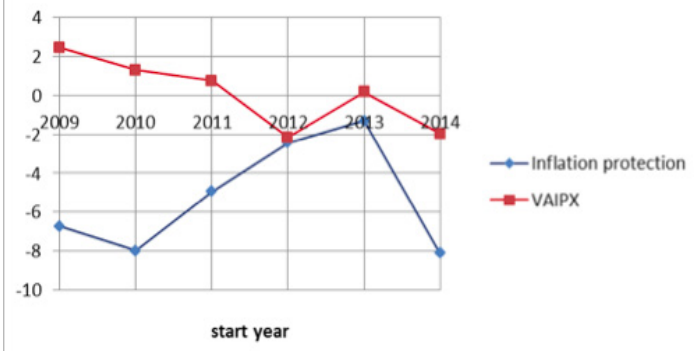


Exhibit 4. Core Real Estate vs VGSLX. VGSLX Returns higher with higher standard deviation of real return and higher maximum % fall in real value.

	Cumulative Real Value		Geometric Avg Return %/yr	
Year	Core Real Estate	VGSLX	Core Real Estate	VGSLX
2015	1	1		
2014	**0.94**	0.96	**6.87**	3.69
2013	**0.85**	0.87	**8.30**	7.30
2012	**0.78**	0.81	**8.53**	7.23
2011	0.74	**0.73**	7.92	**8.22**
2010	0.65	**0.56**	9.13	**12.16**
2009	0.78	**0.37**	4.14	**18.21**
2008	1.13	**0.63**	-1.68	**6.86**
2007	1.09	**0.77**	-1.05	**3.40**
2006	0.96	**0.70**	0.41	**4.00**
2005	0.80	**0.61**	2.25	**5.00**
2004	0.73	**0.48**	2.92	**6.98**
2003	0.70	**0.39**	3.02	**8.14**
2002	0.70	**0.40**	2.78	**7.36**
2001	0.70	**0.35**	2.54	**7.85**
2000	0.65	**0.29**	2.88	**8.56**
1999	0.60	**0.29**	3.22	**8.01**
1998	0.54	**0.32**	3.65	**6.88**
1997	0.47	**0.31**	4.22	**6.82**
1996	0.44	**0.23**	4.42	**7.96**
Std deviation of annual real return %			**8.53**	15.19
Largest cumulative decline in value %			**42.63**	52.10
Largest 3 year decline in value %			28.11	**22.99**

Exhibit 5. Core Real Estate vs VGSLX. Cumulative Real Value. Fraction of Mid 2015 Value. Low means high subsequent return.

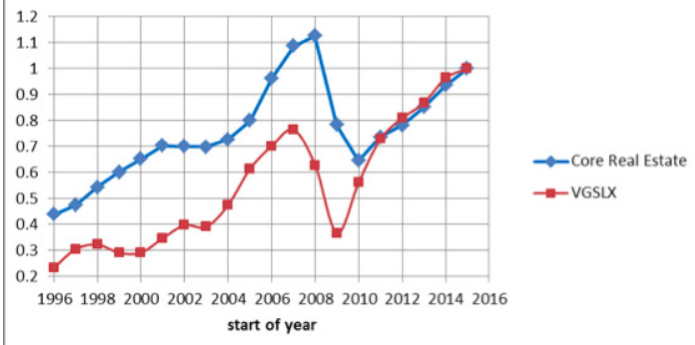
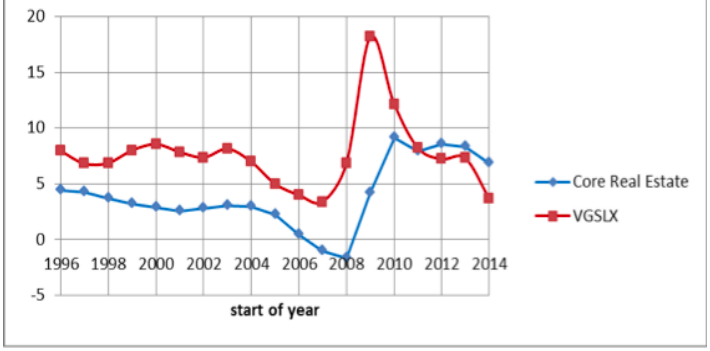
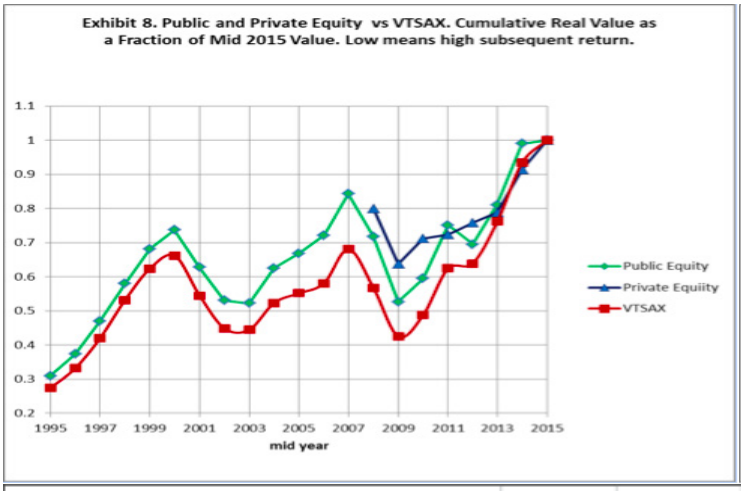


Exhibit 7. Public Equity and Private Equity vs VTSAX. Public Equity returns less than VTSAX by 0.65%/yr with a hair lower std deviation mid 1995 to mid 2015. VTSAX returns higher than either public or private equity with lower std deviation of return than public equity but higher than private equity from 2008 to mid 2015. VTSAX and public equity have almost the same max cumulative fall in real value.

	Cumulative Real Value			Geometric Avg Return %/yr		
Year	Public Equity	Private Equity	VTSAX	Public Equity	Private Equity	VTSAX
2015	1	1	1			
2014	0.99	**0.91**	0.93	0.98	**9.46**	7.04
2013	0.81	0.79	**0.76**	11.05	12.62	**14.57**
2012	0.69	0.76	**0.64**	12.91	9.65	**16.14**
2011	0.75	0.72	**0.62**	7.42	8.42	**12.50**
2010	0.59	0.71	**0.49**	10.95	7.02	**15.45**
2009	0.53	0.64	**0.42**	11.30	7.79	**15.34**
2008	0.72	0.80	**0.57**	4.85	3.23	**8.44**
2007	0.84		**0.68**	2.16		**4.93**
2006	0.72		**0.58**	3.69		**6.22**
2005	0.67		**0.55**	4.11		**6.13**
2004	0.63		**0.52**	4.36		**6.07**
2003	0.52		**0.44**	5.57		**6.99**
2002	0.53		**0.45**	5.00		**6.36**
2001	0.63		**0.54**	3.38		**4.44**
2000	0.74		**0.66**	2.05		**2.79**
1999	0.68		**0.62**	2.43		**2.99**
1998	0.58		**0.53**	3.25		**3.79**
1997	0.47		**0.42**	4.29		**4.94**
1996	0.37		**0.33**	5.33		**5.98**
1995	0.31		**0.27**	6.04		**6.69**
Std deviation of annual real return 2008-2015 %				17.39	**10.89**	16.46
Std deviation of annual real return 1995-2015 %				**15.55**		15.91
Largest cumulative decline in value 2008-2015 %				26.76	**20.36**	25.10
Largest cumulative decline in value 1995-2015 %				**37.580**		37.583

Exhibit 6. Core Real Estate vs VGSLX. Geometric Average Annual Real Rate of Return thru Mid 2015. %/yr.





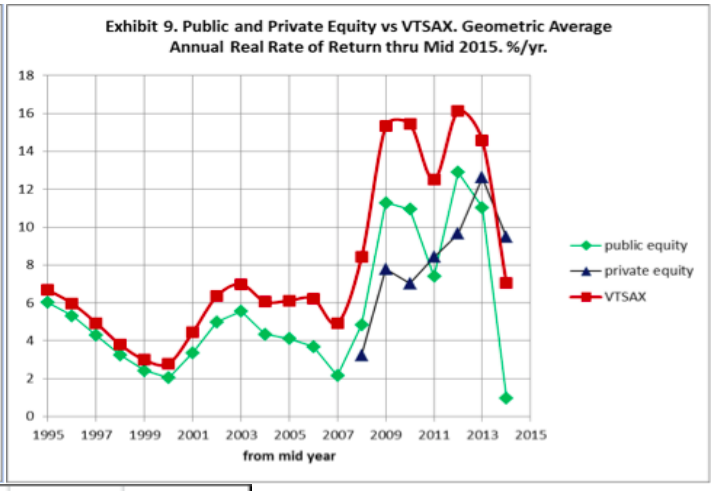


Exhibit 10. Saving from Indexation in $

Pension Component	Start year for Return	Vanguard Return %	Pension Return %	Vanguard Excess	Assets Mid 2015	Saving $ Million
Inflation Protection	2005	1.58	-0.06	1.64	$4,764	$78
Core Real Estate	1996	7.96	4.42	3.54	$3,594	$127
Public Equity	1995	6.69	6.04	0.65	$39,792	$259
Private Equity	2000	2.79	-1.36	4.15	$4,174	$173
Non-Core Real Estate	1995	7.96	4.02	3.94	$3,653	$144
Total Savings						$781

Exhibit 11. Saving from Indexation in perspective

NC Budget 2015	$21.74 billion
# of public school teachers in NC (from NC Education, 2016)	97,308
Average teacher salary (from NC Education, 2016)	$42,557
Population of NC	9.5 million
Savings as a fraction of the NC budget	3.59%
Saving per teacher	$8,031
Saving as fraction of teacher salary	18.87%
Saving per capita	$82

Exhibit A1. From North Carolina Department of State Treasurer
North Carolina Retirement System Quarterly Update 2nd Quarter 2015

Asset Type	Market Value ($000)	Return (expressed in %)				
		1 yr	3 yr	5 yr	10 yr	15 yr
Public Equity	39,792,298	1.1	14.5	13.0	6.3	4.3
Private Equity	4,174,260	9.6	11.1	11.0	8.9	0.8
Non-Core Real Estate	3,652,612	19.6	15.0	14.6	5.2	x
Opportunistic Fixed Income	5,438,220	-0.5	9.7	8.9	x	x
Investment Grade Fixed Income	25,412,562	2.1	2.4	4.8	5.6	6.8
Cash	1,060,871	0.5	x	x	x	x
Inflation Sensitive	4,763,776	-8.0	-1.1	-4.8	2.0	x
Core Real Estate	3,594,338	7.0	10.3	11.9	5.6	x
Multi-Strategy	1,686,815	1.5	11.8	10.3	6.2	x
Total Pension Plan	89,575,752	2.3	9.1	9.5	6.2	5.5

Exhibit A2. North Carolina Retirement System Pension Investment Returns
Annual Returns for Fiscal Years Ending 6/30

Year	Total Pension	Fixed Income	Public Equities	Real Estate	Alter-natives	Private Equity	Hedge Funds	Credit Strat-egies	Inflation Pro-tection	Cash	Non-Core Real Estate	Oppor-tunistic Fixed Income	Multi-Strat-egy
2015	2.3%	2.10%	1.10%	7.00%		9.60%	no report	no report	-8.00%	0.50%	19.60%	-0.50%	1.50%
2014	15.88%	6.04%	24.66%	12.03%		18.27%	6.58%	12.90%	8.21%	0.46%			
2013	9.52%	-0.75%	18.78%	10.90%		5.76%	6.49%	17.45%	-2.95%				
2012	2.21%	11.66%	-5.98%	7.88%	reported	6.55%	-6.52%	0.37%	-10.67%				
2011	18.48%	5.30%	30.76%	18.16%	no	5.24%	6.84%	15.79%	-16.35%				
2010	11.97%	13.20%	14.30%	-16.70%	longer	12.90%	10.30%	8.40%	0.80%				
2009	-14.22%	7.60%	-27.80%	-31.40%		-21.50%	-16.80%						
2008	-2.07%	8.40%	-10.50%	8.70%	7.60%								
2007	14.82%	6.50%	19.90%	15.90%	13.70%								
2006	7.23%	-2.60%	12.60%	25.60%	14.10%								
2005	9.85%	10.26%	9.58%	12.67%	3.59%								
2004	12.01%	-1.08%	23.73%	7.56%	5.57%								
2003	7.56%	15.65%	0.51%	1.97%	-5.83%								
2002	-4.04%	9.96%	-14.67%	0.59%	-20.34%								
2001	-2.04%	11.67%	-12.01%	11.31%	-35.91%								
2000	9.03%	4.50%	12.25%	12.49%	109.43%								
1999	10.74%	1.06%	19.71%	12.96%	26.76%								
1998	19.44%	14.56%	25.63%	16.35%	12.84%								
1997	8.90%	8.46%	28.78%	10.47%	20.19%								
1996	n/a	8.96%	23.87%	6.28%	25.04%								

Exhibit A3. Saving from Indexation in $, Calculated from 5 and 10 Year Trailing Returns

Pension Component		last 5 years to mid 2015				last 10 years to mid 2015			
All returns are average geometric real returns in %/yr	Pension assets mid 2015 $ Million	Vanguard Return	Pension Return	Excess Vanguard Return	Projected Saving $ Million	Vanguard Return	Pension Return	Excess Vanguard Return	Projected Saving $ Million
Public Equity	39,792	15.45	10.97	4.48	1,783	6.13	4.15	1.98	789
Private Equity	4,174	15.45	9.01	6.44	269	6.13	6.70	-0.56	-24
Non-Core Real Estate	3,653	12.16	12.54	-0.38	-14	5.00	3.07	1.93	70
Inflation Sensitive	4,763	1.32	-6.51	7.83	373	1.58	-0.06	1.64	78
Core Real Estate	3,594	12.16	9.89	2.27	82	5.00	3.46	1.53	55
Total	89,576				**2,492**				**969**

Gun Violence in America

A Regression Analysis

Brandt Bruxvoort, Calvin College

Editing by
Gina Rhee and Siddharth Bhaskara

Gun violence is a pressing topic in America. There were 33,636 firearm deaths in 2013 alone. Considerable research has been dedicated to determining the various causes of gun violence. Although gun violence is an endlessly complex phenomenon, scholars have focused on a number of key contributing factors. Among the most contentious of these are gun legislation and gun ownership. Numerous studies have been aimed at discerning the effects of these factors and have frequently come to conflicting results.

Generally, studies have often been directed at discerning the effect of specific gun control laws on firearm deaths. (Kwon and Baack 2005, 534). The effect of "right to carry" laws is a frequent topic of scholarly inquiry, with scholars often concluding that "right to carry" laws lead to fewer firearm deaths. Lott and Mustard (1997), for example, utilize a county-level dataset to argue that gun ownership acts as a deterrent to crime and that right to carry laws reduce gun violence. Mustard (2001) likewise concludes that right to carry laws reduce police deaths. The effect of gun ownership is elsewhere examined in the same light. In these cases the results tend to conflict with the previously mentioned studies, as researchers conclude that higher rates of gun ownership tend to increase gun violence (Duggan 2001; Siegel, Ross, and King III 2013). In some instances a comprehensive look at gun control legislation is attempted. In these studies authors have found that a stronger body of gun control laws is associated with lower rates of gun violence.

The varying results are likely due to a number of factors. First, the effect of individual laws is probably dependent on the general legislative climate (Kwon and Baack 2005, 536). Studies focused on specific laws fail to account for the effect of these other laws in their analysis. Second, gun ownership is particularly difficult to measure and various forms of measurement have been attempted (Siegel, Ross, and King III 2013, 2099). However, most of the literature suggests that stricter gun legislation is a deterrent to gun violence and increased gun ownership is linked to increased gun violence.

Curiously, there is a gap in the literature relating to the intersection of gun ownership and gun legislation. Bice and Hemly (2002) suggest that the passage of stricter gun legislation is linked to increases in gun ownership (262). Scholars have generally failed to take note of this and attempt to separate the effects; indeed, studies are either focused on gun ownership or gun legislation and fail to control for the other in testing effects. Duggan (2001) is the lone exception, although his study only considers right to carry laws rather than gun legislation generally. This paper is an attempt to remedy this literature gap. The aim of this paper is to use regression analysis to determine the specific effect of both gun legislation comprehensively and gun ownership.

Model & Data

The following model contains variables based on the work of Kwon and Baack (2005) with the additional variable of gun ownership. While considerable variation exists in modeling gun violence, a number of core variables have emerged and will be included in the model. Firearm deaths (FD) are here explained by crime levels by state (C), race as percent of the state population that is African-American (R), police expenditures (P), metropolitan population by state (M), unemployment by state (U), gun ownership by state (G), and gun legislation as a dummy variables for both the strongest and weakest states (T10 and B10)

$$FDit=f(Cit, Rit, Pit, Mit, Uit, Git, T10it, B10it)$$

Data were collected for all 50 states from the years 2010-2013 from a variety of scholarly and government sources. See Table 1 for descriptive statistics. Because the data collected take the form of panel data, they will be analyzed using a fixed-effects regression model.

The dependent variable firearm deaths is included as the dependent variable to measure the amount of gun violence in a given state. Although it is not universally chosen as the dependent variable in the literature, it is commonly chosen due to its measurability and because gun legislation is primarily aimed at preventing gun deaths. It is operationalized here as the number of age-adjusted firearm deaths per 100,000. Data were collected from the National Vital Statistics Report compiled by the Centers for Disease Control

For independent variables the model first includes crime levels. Crime-levels are a common variable throughout the literature, and are linked to gun violence by multiple models (Kwon & Baack 2005, 538; Lott and Mustard 1997; Mustard 2001, 641). It is assumed that guns are a common component of many crimes and therefore will tend to increase gun deaths as crime also increases. For this reason crime rates are hypothesized to have a positive coefficient. Crime level data was operationalized as the rate of violent crimes per 100,000 as reported in the Uniform Crime Reports (Table 4) published annually by the FBI.

Although there is no reason to think that race itself is a causal element in gun violence, race is linked to wide variety of complicated socioeconomic factors that do influence gun violence. Research has demonstrated that race both influences the likelihood of being a victim of gun violence (Felson and Painter-Davis 2012) and that race is correlated with other factors leading to crime (Eitle and Turner 2003). Furthermore, previous models have used race as a variable (Kwon and Baack 2005, 538; Lott and Mustard 1997, 15). It is therefore included in this model as a general measure of these factors and is hypothesized to relate positively to firearm deaths. Race was operationalized as the percentage of African-American residents per state, and was collected from American FactFinder (United States

Census Bureau).

An increased police presence is assumed to have a deterrent effect on crime and illegal gun usage and to consequently reduce gun violence (Schargrodsky and Tella 2004, Marvell and Moody 1996, Mustard 2001). Here an increased police presence is hypothesized to have a negative coefficient. Police presence was operationalized here as the percentage of the state population working as full-time law enforcement employees, in the same manner as Kwon & Baack (2005). Data were collected from Table 77 of the FBI's Uniform Crime Reports.

Metropolitan areas is a complex variable with multiple possible interpretations (Kwon and Baack 2005, 540). Some research has suggested that metropolitan areas have higher crime rates and should thus be assumed to also contribute to higher levels of gun violence (Pack 1998, 27). Other studies, however, link higher gun ownership with rural areas, thus implying lower gun violence with more people in a metropolitan area (Duggan 2001, 1090). Given the compelling theoretical case for higher crime in cities influencing gun violence, it is hypothesized that metropolitan areas will have a positive coefficient. Here the variable is operationalized as the percentage of the state residing in a metropolitan area. Data were only available for 2010 and 2013, so 2011 and 2012 data were estimated using a linear interpretation of the change from 2010 to 2013. 2010 data were sourced from the Census Bureau's "Statistical Abstract of the United States: 2012," and 2013 data from the Henry J. Kaiser Family Foundation, which bases its statistics on Census Bureau data.

Unemployment is linked to violence in a number of studies (Fallahi, Hamed, and Gabriel 2012) and is included in several models of gun violence (Kwon and Baack 2005, 539; Lott and Mustard 1997, 16). Here it is assumed that higher unemployment provides a greater incentive to utilize criminal methods for income, and consequently increases gun use. Therefore it is hypothesized to relate positively to firearm deaths. Unemployment is measured as the unemployed percentage of the labor force as reported by the Bureau of Labor Statistics.

Gun ownership is one of two variables not included in Kwon and Baack's (2005) base model. Here it is included to test for its effect as distinct from gun legislation. The literature on gun ownership is self-contradictory. Studies examining right-to-carry laws have concluded that the higher ownership encouraged by the laws acts as a deterrent to gun violence (Lott and Mustard 1997, 59; Mustard 2001). Alternately, a number of studies have generally linked gun ownership to higher rates of violence (Duggan 2001, 1112; Siegel, Ross, and King III, 2013, 2103). In this model gun ownership is hypothesized to have a positive coefficient. As previously mentioned, gun ownership is difficult to measure due to the lack of a reliable and direct measurement. Here gun ownership was operationalized using a common proxy, the percentage of suicides committed by firearm (or FS/S) (Siegel, Ross, and King III 2013). Data for the proxy werer gathered from the Web-Based Injury Statistics Query and Reporting Systems from the Centers for Disease Control.

Like gun ownership, gun legislation's effect on firearm deaths is a topic contested in the literature. Papers on the effects of specific laws have concluded that stricter laws increase gun violence (Lott and Mustard 1997, Mustard 2001), while others have linked strict legislation with decreased gun violence (Anestis and Anestis 2015; Duggan 2001; Fleeger, Lee, Monuteaux, Hemenway, and Mannix 2013; Siegel, Ross, and King III 2013; Simonetti, Rowhani-Rahbar, Mills, Young, and Rivara 2015). As in Kwon and Baack's (2005) base model, this paper also adapts the approach of looking state-level gun laws comprehensively rather than individually. Stricter gun control legislation is here hypothesized to relate negatively to firearm deaths. Kwon and Baack's (2005) methodology for operationalizing the variable was also adopted here. Data were collected from the Law Center to Prevent Gun Violence and The Brady Campaign to Prevent Gun Violence, both of which rank each state by the strength of its gun legislation. Rather than using the scores assigned by the Law Center, this paper simply includes two dummy variables, one for being ranked in the top ten and another for being ranked in the bottom ten. This approach has the advantage of not relying on the particular scores assigned, thus distancing the results from any potential biases in the rankings. Furthermore, the approach does not assume a linear relationship between firearm deaths and each state's score. Comparative data for the top and bottom ten are provided in Table 2.

Variation also exists in the literature concerning the functional form for modeling gun violence. Lott and Mustard and Duggan utilize a double-log form, while Kwon and Baack (2005) and Siegel, Ross, and King III (2013) utilize a linear relationship. Here both forms will be investigated in an attempt to find the most appropriate form and to test the resilience of the estimates to changes in functional form.

Results

The model was first estimated using a fixed effects panel estimation (regression 1-F), then compared to the corresponding random effects regression (regression 1-R) with a Hausman test. Although regression 1-R had a significantly higher R2 and allowed for the preferred method of data collection for gun ownership, the Hausman test demonstrated significant differences in the coefficients and ruled out the possibility of using regression 1-R. Table 3 describes the results of regression 1-F.

With a low R2 of 0.2847 and only three statistically significant variables Regression 1-F fails to provide much information. Although the coefficients for both T10 and B10 lend support to the hypothesis that strong gun legislation decreases firearm deaths, neither have enough statistical significance for this conclusion to be meaningful. However, O does have a coefficient that is positive as expected, statistically significant, and clinically large. Its coefficient of 10.102 implies that even a one percentage point increase in the percentage of residents owning guns in a particular state will result in an additional 10 firearm deaths per 100,000 that year. Problematically, U has

a negative coefficient, implying that as unemployment increases firearm deaths would decrease. This conclusion is both illogical and contradicted by previous research. Consequently, the coefficient for U casts doubt on all the conclusions in regression 1-F and suggests that there may be a specification error due to an omitted variable. Ultimately, due to the lack of statistical significance and potential specification problems, the functional form of regression 1-F cannot be used to make any conclusions about the hypotheses or the nature of gun violence.

Regression 2-F presents the same regression in log-log form, reported here in Table 4. A random-effects regression (regression 2-R) was also conducted in double-log form. However, a Hausman test again ruled out the use of random effects and invalidated regression 2-R. Additionally, a Breusch-Pagan test of 2-F revealed significant heteroscedasticity. A generalized least squares regression, regression 2-G, corrected for the effects of heteroscedasticity and its results are presented in Table 5. Since the standard errors in 2-F are biased, I will limit the discussion to the results of 2-G.

The results from regression 2-G were more statistically significant than the results from 1-F, suggesting that a significant amount of the variation in ln(FD) was explained by the model. Significant variables included ln(B10), ln(C), ln(U) and ln(O). The negative coefficient on ln(B10) contradicts the idea that gun control laws reduce firearm deaths and in fact implies the opposite, although the effect implied in the model is small. The natural log of crime, however, gave the expected result of having a positive effect on firearm deaths, although the effect is small. The model predicts that it would take a five percent increase in violent crimes to create a one percent increase in firearm deaths. The 0.679 coefficient on ownership does imply, as expected, that firearm deaths increase by over half a percent for a corresponding one percent change in ownership. On the other hand, ln(U) again has a problematic interpretation. Its negative coefficient again implies that even a ten percent increase in unemployment could decrease gun deaths by one percent. Again, this result raises doubts about the specification of the model and suggests the possibility of an omitted variable. Furthermore, ln(T10) (having a strong body of gun legislation) returned an astronomical p value of 0.977, implying that it is not merely insignificant but very likely a zero coefficient of in reality.

Discussion & Conclusion

Overall, the models considered in this p per contributed some reliable knowledge to the area of gun violence. Of the functional forms tested, the double-log form provided better statistical results. The disparity between the two forms suggests that there is not a robust relationship between the data here and the relationships tested. Despite the higher quality of the double-log form, both functional forms showed problems of statistical significance, and tests hinted at the possibility of deeper statistical problems. The most important of these is the unexpected and implausible negative coefficient on both U and ln(U). The most likely cause for this is an omitted variable. With the distinct possibility of an omitted variable, there is a strong possibility of bias in the other variables. Consequently, the results in this paper should not be used to draw conclusions about the nature of gun violence in America.

Further research into this subject matter should focus on providing a better dataset. The significant variation within states of many of the variables suggests that county-level data may allow for more productive research, as suggested by Lott & Mustard (1997, 5). More in-depth cross sectional data might also be paired with a longer time frame to further evaluate gun violence by increasing the sample size and allowing to better observe changes over time. Finally, there is still much to be learned about the relationship between gun ownership and gun laws. If the results in this paper correctly suggest that having lax gun legislation is indeed associated with fewer gun deaths, there are still questions as to the mechanism for that decrease. It may be that Lott and Mustard (1997) are correct that the lax laws enable citizens to deter crime through self-defense. However, Bice and Hemley (2002) could also be correct to suggest that since gun laws increase ownership, lax laws may actually have an effect of gun violence by decreasing gun ownership.

Table 1: Descriptive Statistics

Variable	Mean	Median	Standard Dev.	Min	Max
Firearm Deaths (FD)[9,10]	11.27	11.1	3.99	2.60	20.40
Crime (C)[9]	355.32	331.8	131.00	121.10	660.60
Race (R)	10.75%	7.88%	0.095	0.43%	37.40%
Police (P)	0.32%	0.31%	0.0006	0.21%	0.59%
Metropolitan (M)	73.98%	75.24%	0.174	29.66 %	100%
Unemployment (U)	7.74%	7.80%	0.020	2.90%	13.50%
Gun Ownership (O)[11]	51.79%	52.70%	0.128	16.4%	75.27%

Table 2: Comparative Statistics for Top and Bottom Legislative States

Variable	Top 10		Bottom 10	
	Mean	Standard Deviation	Mean	Standard Deviation
Firearm Deaths (FD)[9,10]	11.27	2.684	14.31	3.5
Crime (C)[9]	355.32	90.791	321.04	149.61
Race (R)	10.75%	0.076	7.91%	0.071
Police (P)	0.32%	0.0006	0.32%	0.0007
Metropolitan (M)	73.98%	0.087	56.85%	0.173
Unemployment (U)	7.74%	0.017	6.96%	0.018
Gun Ownership (O)[11]	32.17%	0.100	60.71%	0.068

TABLE 3

RESULTS FOR REGRESSION 1-F (DEPENDENT VARIABLE: FIREARM DEATHS PER 100,000)

	Regression Coefficients					
Variable	*Coefficient*	*Lower (95%)*	*Upper (95%)*	*Standard Error*	*t*	*p>|t|*
T10	0.164	-0.620	0.948	0.397	0.410	0.680
B10	-0.364	-0.728	0.000	0.184	-1.980	0.050
C	0.006	0.000	0.012	0.003	2.070	0.041
R	31.237	-75.689	138.164	54.090	0.580	0.565
P	-320.121	-960.932	320.689	324.164	-0.990	0.325
M	0.766	-6.363	7.895	3.607	0.210	0.832
U	-20.762	-35.707	-5.817	7.560	-2.75	0.007
O	10.102	6.061	14.144	2.044	4.940	0.000
Constant	2.613	-9.321	14.547	6.037	0.430	0.666

TABLE 4

RESULTS FOR REGRESSION 2-F (DEPENDENT VARIABLE: NATURAL LOG OF FIREARM DEATHS PER 100,000)

	Regression Coefficients					
Variable	*Coefficient*	*Lower (95%)*	*Upper (95%)*	*Standard Error*	*t*	*p>|t*
ln(T10)	0.001	-0.077	0. 079	0.040	0.030	0.977
ln(B10)	-0.033	-0.069	0.003	0.018	-1.82	0.070
ln(C)	0.199	-0.023	0.421	0.112	1.770	0.079
ln(R)	0.025	-0.169	0.219	0.098	0.250	0.799
ln(P)	-0.119	-0.339	0.100	0.111	-1.07	0.285
ln(M)	-0.030	-0.495	0.435	0.235	-0.13	0.899
ln(U)	-0.118	-0.228	-0.009	0.055	-2.14	0.034
ln(O)	0.679	0.497	0.861	0.092	7.380	0.000
Constant	0.734	-1.396	2.865	1.078	0.680	0.497

TABLE 5

RESULTS FOR REGRESSION 2-G (DEPENDENT VARIABLE: NATURAL LOG OF FIREARM DEATHS PER 100,000)

	Regression Coefficients					
Variable	*Coefficient*	*Lower (95%)*	*Upper (95%)*	*Standard Error*	*z*	*p>|z|*
ln(T10)	0.001	-0.064	0.066	0.033	0.030	0.973
ln(B10)	-0.033	-0.063	-0.003	0.015	-2.16	0.030
ln(C)	0.199	0.013	0.384	0.095	2.100	0.036
ln(R)	0.025	-0.137	0.187	0.083	0.300	0.762
ln(P)	-0.119	-0.303	0.064	0.094	-1.27	0.203
ln(M)	-0.030	-0.418	0.358	0.198	-0.15	0.880
ln(U)	-0.118	-0.210	-0.027	0.047	-2.54	0.011
ln(O)	0.679	0.527	0.831	0.078	8.760	0.000
Constant	1.181	1.250	0.210	-0.666	3.028	0.942

The Quest for Survival

Modeling Congressional Voting Behavior with Campaign Contributions

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The business world functions within the realm of laws. The laws that set the limits and the framework of their business operations are promulgated by the legislature branch. But the relationship between the legislature and business sector is not unidirectional. The business world has a tremendous influence over the legislature as well, though the metrics to quantifying such influence is not as apparent. Regarded largely as hideous and conspiratorial, the nature of monetary influence on politics has not always been viewed in a positive light. But in reality, money remains as one of the strongest mechanisms that influence our politics. It is far-fetched to say that the politicians should not be influenced by monetary incentives. According to the Center for Responsive Politics, candidates for the House of Representatives spent on average of 1.7 million dollars for the 2014 congressional cycle and those for the Senate a staggering 3.7 million. However, what remains elusive is where that much money comes from and why it exists.

This question has indeed been the underlying motivation for many scholars of the literature on campaign finance. In fact, scholarly attempts to uncover the relationship between money and political decision-making date back to the 1970s when the Federal Election Campaign Act provided for open access to campaign finance data. A brief historical overview reveals how a bulk of empirical research on the influence of money into politics would not have been possible without the Federal Election Commission. For one of the consequences of the Federal Election Campaign Act of 1971, the Federal Election Commission has been established and has since officially compiled all individual contributions above $200 and institutional donations made by Political Action Committees (PACs). This paved the way for the research community to retrieve and use government-approved data to substantiate their theoretical foundations with empirical substance. In addition to the Federal Election Commission, scholarly attempts to gather campaign finance records that predate the 1970s were made by Alexander (1972), Alexander and Fisher (1974), and Alexander and Jones (1971). With increased efforts to solidify raw data on campaign finance records, other subsequent studies drew interdependent relationships between the business sector and the American politics using both theoretical and empirical models (Kau and Rubin (1978), Stigler (1971), Peltzman (1976), and Posner (1974)). Such initial empirical research and data compilation in the early 1970s pioneered the field of the modern American literature on campaign finance that we know today.

Despite its history over five decades, however, past studies on campaign finance have long been largely divided over the efficacy of and the motivation behind such monetary contributions. In their meta- analyses of 35 previous empirical studies performed with 357 models, Roscoe and Jenkins (2005) conclude that one-third of studies they tested had statistically significant influence of campaign contributions on voting behavior. Also, the motivations underlying campaign donations each model assumes vary. Snyder (1990) views campaign donations as long-term investment to foster favorable relationships. For some other arguments on such motivations behind campaign donations, Ansolabehere et al. (2002) argue that campaign donations are mere expressions of political interest and personal consumption. While some like Clawson, et al. (1996) argue that it is rather used as gifts to encourage mutually beneficial agreement, others like McChesney (1997) conclude that campaign donations are used as an excuse for coercion.

In order to expound upon the American campaign finance system, understanding the breakdown of how money gets injected into the political system should come first. As Burnstein (2003) mentions, there are two main ways interest groups have historically influenced politics through money: campaign contributions and lobbying. The first way for money to be injected into politics is colloquially termed hard money where donations are directly made to a political candidate. This type of contributions mainly come from individuals or political/ action committees, while corporations are not allowed to fund them directly. A political action committee (PAC) is a separate private organization founded by interest groups like business, labor unions, and other types of organizations. Boeing, for example, has its own political action committee (BPAC) that is responsible for handling government affairs. There are limits officially set forth by the Federal Election Commission on the

maximum amount individuals and PACs can donate with hard money. On the other hand, there is soft money that not just come from individuals but also directly from institutions like corporations. Soft money is contributed to a political party and while there are no limits, the donations cannot be directly made to individual congressman's campaigns. Because of how unlimited use of soft money engendered the race for fundraising in congressional campaigns, the Bipartisan Campaign Act of 2002 banned contributions made by interest groups and national political party committees that are not subject to federal limits.

Even within the realm of hard money, the practice of defining the limits on campaign contributions is still ongoing. On April 2nd 2014, the Supreme Court's divisive 5-4 decision that ruled the Nixon Administration-era limit on campaign contributions unconstitutional sparked major controversies, a decision that still generates frenzy up to this day. While the reasoning was that the limit violated Freedom of Speech, some critics claim that the decision was to attract more money into politics. This decision made major changes to the Amendment to the Federal Election Campaign Act in 1974 that had previously defined the two types of limits on campaign contributions. The base limit – the monetary restraint on how much a contributor can donate to a candidate or a committee – remained at $2,600 per donor. The aggregate limit, on the other hand, was ruled unconstitutional. Irrespective of the debate, the current limit on campaign contributions remains set to $2,600 for individual contributions and $5,000 for PAC contributions.

This paper is an empirical test of how corporations influence congressional behavior. Specifically, I test the causality of hard money donations, on to what extent the amount of hard money determines congressional behavior and vice versa. I focus on a single type of a Congressional bill termed the U.S. Export-Import Bank Reauthorization Act. Created under the FDR Administration in 1934, the U.S. Export-Import bank is a federal export credit agency that provides seven mechanisms of financial assistance programs to American export companies. But the bulk of their lending activities is under three main lending programs: i) fixed-rate direct loans to foreign buyers of U.S. export goods, ii) loan guarantees to domestic manufacturers at low fees, and iii) export credit insurance (ECI) for smaller firms. Direct loans are provided to foreign buyers of U.S. exporters at a cheaper interest rate than that of the market. ECI's are for smaller firms that help hedge their future financial loss if the foreign buyer fails to pay for the goods they ordered. The three lending schemes provide an interesting viewpoint on the nature of the debate surrounding the bank. Recent arguments made by the U.S. Export-Import Bank to counter some of its opponents are on how it fulfills its mission to address the needs of smaller companies in exporting their goods through ECI. However, the focus of the debate should rather be on the loan guarantees because in terms of magnitude loan guarantees account for a bulk of taxpayer's money. In FY 2013, 74% of the direct loans provided to international buyers were for purchasing heavy-machinery products, which suggests that the companies that supply such goods need much capital to produce them in the first place and therefore cannot be small business with revenues less than 10 million dollars. To sum up, the majority of the type of loan authorizations (ECI) that the Bank claims is reflective of its commitment to focusing on the smaller companies belies the fact of how the majority of money goes to somewhere else (loan guarantees, direct loans) to fund bigger companies, as will be mentioned specifically in subsequent charts.

Charts 1 and 2 provide the breakdown of the three main product types for FY2013. Here, Export Credit Insurance programs account for a majority 81% of the total number of the bank's loan activities, but Chart 2 shows that loan guarantees account for 55% of the total dollar amount of the bank's authorizations. This suggests that while most number of authorizations are through ECIs targeted towards small companies, the majority portion of the money was spent on loan guarantees. Now, looking into Chart 3, we see that the top 10 companies received 97% of the total dollar amount of loan guarantees in 2013. The charts tell a revealing story. While 3,097 small businesses received 20%, the top nine companies in Chart 3 received at least 53% of the total amount of authorizations in FY2013. These three charts combined show that a small number of companies in the manufacturing and aerospace industry reaps the bulk of taxpayer's money through loan guarantees, with Boeing leading the pack. In Chart 4, we can also see that the bulk of the bank's total authorization amount is made to big companies that require long term production cycle and investment capital such as aircraft, agricultural equipment, or steel.

On the other hand, campaign finance data suggest that these major beneficiaries shown in Chart 3 are also big influencers on the political sphere. Chart 5 is the percentage of incumbent Republicans and Democrats who received campaign contributions from the top four companies listed above in Chart 3 and 4: Boeing, General Electric, Bechtel, and Caterpillar. For the congressional votes on the U.S. Export Import Bank held in the years 1992, 1997, and 2012, these four major companies contributed to increasing percentage of incumbent congressmen as years passed, reaching the peak of 95% of the incumbent congressmen from both Republican and Democratic Parties in 2012. Given that the amount of donations do not vary much among congressmen (low standard deviation) within $5,000 limit for PAC contributions, this suggests that the companies spend more money than ever as campaign donations. Also, according to the Center for Responsive Politics, two of the top four beneficiaries (Boeing and General Electric) in Chart 3 are what the Center classifies as "Heavy Hitters" – groups that lobby and spend big, with large sums donated to candidates, parties and leadership PACs, while Caterpillar and Bechtel spent $800,000 and $720,000 of campaign contributions for every congressional cycle since 2008.

What remains to ask then is why such big companies that have been the beneficiaries of the Bank spend ever more money on campaign donations. This, I argue, is due to the political climate surrounding the bank that it should be dismantled. Chart 6 shows the percentage of Republicans and Democrats that voted in favor of the reauthorization of the Bank. Though there is no clear trend of the two Parties, one important point to note is how the percentage for Republicans plummeted in 2012 after their unanimous support for the bank in 2006. This was partly a result of the recent accusations that began among the Republican incumbents including Jeb Hensarling that the Bank is a manifestation of crony capitalism that supports only a few big companies. Regardless of the implications of the dispute, what remains true is that the greater

number of Republicans have started voting against the bank and the top beneficiaries of the bank are contributing ever more heavily to the incumbents. This is where I hypothesize that the increasing magnitude of campaign donations is due to the top beneficiaries' alarming need to encourage congressmen to vote for the bank again.

Uncovering such motivations behind increased spending on campaign donations Chart 5 is what I attempt to focus more here. In Chart 5, incumbent congressmen received campaign contributions in record proportions as time progressed, but the overall voting record in 2012 reveals a counterintuitive result, with a steep drop for the Republicans after their unanimous support in 2006. This suggests that campaign donations the companies made did not attract many votes. This has two implications. The first is whether the campaign donations increased because the companies expected that more congressmen will vote against the bank or that despite the increased campaign donations, the donations made by businesses that were against the U.S. Export Bank began to eclipse those that were in favor. The two implications have two diverging interpretations. The first implication suggests the endogenous nature of the campaign contributions in that they are not fully independently determined and that there is a potential for simultaneous bias. As I will elaborate further in the next section, simultaneous bias comes in when the probability of voting and the amount of campaign donations are simultaneously determined. The second implication suggests that the congressional voting behavior is endogenously determined by exogenous campaign contributions provided by conflicting businesses. In other words, the amount of campaign donations the companies make is not determined by congressional voting behavior. I will address both of these implications in the section on model specification.

Throughout my analysis of the debate surrounding the U.S. Export Import Bank, I attempt to address the issue of simultaneity bias in my empirical research into the campaign donations model. The section on the Literature Review provides past research attempts done by scholars in the field to address such bias. The following section on model specification and methodology will address the assumptions I made in making my model and the model itself. The last section concludes.

II. Literature Review

In this section, I briefly provide some theoretical framework that sets up the model I wish to specify in the next section. I will first discuss the motivation behind an incumbent congressman's interest in receiving campaign contributions. Then, I provide some of the outside forces that potentially influence congressional voting behavior. Last, I provide an explanation of how 2SLS could be used as I treat campaign donations to be endogenous.

In the realm of Political Economy, it has been commonly shared among scholars as an assumption that congressmen seek reelection. While some others have argued that the goals are rather more multidimensional, from internal power in Washington Politics to legislating a good policy (Fenno 1973, Kingdon 1989), these remain trivial compared to the substantial amount of past research done on under the reelection assumption (Mayhew 1974, Parker 1992). This entails several important theoretical conclusions. The first is that we can predict Congressional voting patterns only by looking at the factors that could influence the members' reelection results. The second is that vote-buying hypothesis is more consistent with the positive relationship between campaign contributions and congressional voting behavior than ideological sorting hypothesis. If the congressmen were influenced by factors that could enhance their candidacy in the next Congressional cycle, then they will stick with whatever could provide them the greatest support for reelection (known as vote buying hypothesis) rather than stick with their own original agenda (Ideological sorting).

Indeed, some scholars have empirically shown vote-buying hypothesis to be more accurate for junior congressmen (Bronars and Lott 1997). Given that reelection rates among incumbent congressmen historically remained in the 80~90 percent range, it is hard to disregard reelection out of the question of what motivates an incumbent congressman's voting behavior. On the other hand, because incumbents receive on average 2.4 times more donations than the challengers according to the Center of Responsive Politics, it suggests that on average incumbents receive more and get reelected more. Therefore I assume maximizing campaign contributions to be one of the primary goals congressmen have before running for reelection.

Now looking into some of the literature on campaign finance, a number scholars have made various approaches to answering the unidirectional causality of the campaign donations and their effect on congressional voting behavior. Florina (1974), Kingdon (1973), Matthews and Stimson (1975), and Silberman and Durden (1976) each provide a unidirectional equation of donations affecting voting behavior. Ansolabehere et al. (2003) presents a collection of past studies and concludes that campaign contributions are significant for a quarter of the studies they analyzed. In his research into the financial services industry, Stratmann (2002) uses similar legislation on the Glass-Steagall Act to isolate the effect of money on votes by using member-fixed effects. Then using his methodology, he concludes that donations by financial services firms do affect congressional voting. On the other hand, other papers revealed the other direction of campaign finance dynamics. For example, approaches have been made on how the personal characteristics of legislators – senior vs junior, party affiliation, committee membership – affect the amount of campaign contributions they bring in (Gokcekus and Barth (2006), Gokcekus, Knowles, and Tower (2004)). But these studies, along with most of the rest, have not yet developed a bi-directional empirical model of what determines or changes politicians' voting behavior. Specifically, causality seems to work in both directions and campaign donations do influence voting behavior as much as how votes could determine how much contributions the congressmen will receive in the next congressional term.

This then brings us to the next step of the problems inherent in campaign finance literature. Up to date, the challenge in the campaign contribution literature has been to identify this dynamic interplay of economic agents and address the chronic methodological issue of simultaneous-equation and endogeneity biases. Simultaneous-equation bias occurs when an ordinary least squares method becomes a part of another system of simultaneous regression equations. It is when the independent vari-

ables in two simultaneous linear equations are correlated with the error terms. The error terms become correlated with the independent variables because the independent variables are now dependent on the error terms. Econometricians attempted to solve this problem of simultaneous causality bias (independent variable causing dependent variable and vice versa) using two-stage least squares equations with instrumental variables regression. This method is to choose some set of instrumental variables that is correlated with the original independent variable while uncorrelated with the original error term. Especially for campaign finance literature, this method has largely been used because campaign contributions and voting behavior cause each other that generate simultaneous equations bias (Jacobson 1978, 1980, 1985, 1990; Green and Krasno, 1988; Gerber 1998). Endogeneity bias occurs when there is a correlation between the parameter and the error term of the equation. Omitted variable bias is an example of an endogeneity bias in that it is due to uncontrolled confounding variable.
Existing theories and research have been conducted on how these three economic agents interact using simultaneous equations models. In the 1980s before Chappell (1983) first introduces simultaneous equations models to campaign finance literature, Kau, Keenan, and Rubin (1982) employ simultaneous equations to draw the relationships between these agents in a given time frame. Here, they use three simultaneous equations in which a vote on a bill is determined by a vector of campaign contributions by different interest groups, while the vector of campaign contributions is determined by the congressman's voting behavior. They use two stage simultaneous estimation procedures with instrumental variables to solve these equations, but do not report what they used as exogenous/ instrumental variables for their first- stage equation.

As many scholars in the literature agree, there has not yet been a systematic way to capture the endogenous nature of campaign contributions. Despite such empirical constraints however, Gordon (2005) suggests there are three techniques that could be used. One is a two-stage least squares analysis (2SLS), while others are Simultaneous Probit-Tobit (SPT) procedure and two-stage limited dependent variable (2SLDV) analysis. Among them, two-stage least squares method has been largely used among scholars. It takes advantage of instrumental variables that are used as a fixed regressor in the second stage. Then any correlation between the instrumental variables and the dependent variable in the second equation is purely due to the effect of the independent variables in the second equation (Chappell 1981; Pindyck and Rubinfeld 1981). Hence, I will employ 2SLS methods to get the coefficients I need if simultaneity comes in.

In order to employ 2SLS methods, choosing the right instrumental variables is essential in generating the right model. However, despite many efforts to include the right instrumental variables (Hausman, J.A. 1983, Kelejian 1973) for 2SLS models, no consistent methodology of finding the right instrumental variables were developed in the general construct of campaign finance literature. This issue is what I wish to address specifically in the case of the U.S. Export-Import Bank by choosing instrumental variables that are correlated with campaign contributions but not with the congressman's voting behavior.

Throughout this paper, I hope to add to the existing literature on campaign finance the idea that campaign contributions are used as a rather more strategic means for the business community to win over congressional votes. In addition, I welcome any future endeavors to systemize the way to find instrumental variables that capture campaign donations.

III. Methodology

Before I dive in to the specific system of equations, there are some assumptions I wish to clarify. I assume congressional behavior to be simultaneously determined by the amount of campaign contributions given between the last vote and the current vote. I am therefore assuming that the only the most recent contributions affect voting behavior and the donations that precede the previous election cycles irrelevant. Just because the previous donations did help the congressman win a previous election does not mean they should help finance the congressmen's election at time t. In addition, the business community provides campaign contributions and the amount indicates the certainty to which the business believes the congressman will side with the donor. Hence, if a company donates the maximum limit to a congressional member for every congressional term, this in turn means that the company fully entrusts the congressman to side with the company's priorities. The next assumption to add is that the congressman has always been interested in getting reelected and tried to get as much donations as possible to support his campaign.

I assume a congressman's voting behavior on four main inputs. 1. Amount of Campaign contributions made from her last vote on the bank to now, 2. Her vote on the most recent previous bill, 3. Party Alliance, and 4. Federal funds rate. As many empirical papers on other Congressional bills corroborate, the factors that determine a congressman's political stance such as the state she represents, the year she first got elected, or her initial voting behavior on the bill remain the same throughout the course of her consecutive votes on the Reauthorization bill, so I capture her constant factors into her previous_vote variable. Second, I expect the amount of campaign contributions made by the selected four companies to have a positive effect on the congressman's voting behavior, since these companies have been the biggest beneficiaries of the bank. I normalize it to the total amount of campaign contributions the congressman receives in between the last vote and the current vote because there is a possibility for the opponents of the Ex-Im Bank to contribute more to the congressmen. Third, I expect party alliance at time t to affect congressmen's vote at time t because what the party dictates at the time of the vote could influence the congressman's stance on the bill. Lastly, I expect the congressman to vote yes on the bank's existence in times of macroeconomic boom, because the market interest rates are high in bullish markets and there are smaller companies who cannot afford loans from the market with such high interest rates. So I add the variable on the fed funds rate to control for the situation where congressional behavior is affected by smaller companies other than the four companies I chose above because the smaller companies are more affected by the smaller companies who are more susceptible to high interest rates. Then I get the following equation:

$$p(vote)_i^t = \frac{e^{x_i^t}}{1+e^{x_i^t}}, \text{ where}$$

$$X_i^t = \beta_0 + \beta_1 \frac{CC_i^t}{TC_i^t} + \beta_2 prev\_vote_i^{t-1} + \beta_3 party_i^t + \beta_4 fedfunds\_rate_i^t + u_i^t \quad (1)$$

Here, the amount is $\frac{CC_i^t}{TC_i^t}$ defined as the normalized amount of campaign contributions congressman *i* receives in time *t* by the top beneficiaries of the bank (Boeing, Bechtel, General Electric, Caterpillar) divided by the total amount of campaign contributions that congressman i received at the congressional term t. I normalize the amount of campaign contributions the four companies made to total amount of campaign contributions because even if the amount were high, their donations would not be a huge factor to the overall amount of donations if there were significant amount of donations made by the opposing companies of the bank. Note that time t does not refer to a specific congressional term, but the timeframe between the last vote and the current vote. Since the reauthorization vote was held in five years on average, there are on average three congressional terms in between each votes, which means that the donors can donate at most six times with a limit of $5,000 because there are primary & general elections for each congressional term.

Now that I have taken care of the equation for the congressman's voting behavior, it is time to look at the amount of campaign contributions made by the four. I assume the contributions to not just be a mechanism to influence congressmen to vote yes on the coming bill but also to retaliate against those who stopped voting yes and vote no. Consistent with the Vote Buying Hypothesis, I treat the amount of campaign contributions to be endogenously determined by current voting behavior. This is where the endogeneity bias comes in. As a general rule, if a variable is endogenous, it will be correlated with the disturbance term so the OLS estimates are now biased. Looking at equation (1) means cov(CC_i^t, u_i^t) > 0, which violates the assumption of OLS regression. The theoretical assumptions that the OLS regression make are that the coefficients should be unbiased and the stochastic error not correlated with any of the independent terms. The violation comes from the following equation.

$$CC_i^t =: \sum_j CC_{\{j,i\}}^t, and$$

$$CC_i^t = \gamma_0 + \gamma_1 vote_i^t + \gamma_2 prev\_vote_i^{t-1} + \gamma_3 party_i^t + \gamma_4 fedfunds\_rate_i^t + v_i^t \quad (2)$$

Here, $CC_{\{j,i\}}^t$ refers to the contribution made to congressman i at time t by company j, so we need to sum all the contributions made by the companies to see how much the companies that support the Ex- Im Bank donate. If we plug in equation (2) to (1), then we get how the amount of campaign contributions is proportional to the error term u^t_i and this is how we get cov($vote_i^t, u_i^t$) > 02. This poses a problem and our standard OLS multivariate regression will involve a bias. Therefore its coefficients are going to be wrong because they are providing biased estimates. This is why I decide to use the two-stage least squares method with instrumental variable approach in lieu of the voteti variable in equation (2). I expect the amount of previous campaign contributions ($CC_{j,i}^{t-1}$) the company donates to congressman i to be uncorrelated with the congressional behavior on the bank reauthorization vote but is correlated with the 2 Proof provided in Appendix I. amount of campaign contributions donated in at time t, so that will be used as the instrumental variable of my interest. Note that this comes from my assumption that the congressmen's voting behavior is determined only by the most recent campaign donations, not the ones that go before the previous election cycle. This is because the amount of campaign contributions in the past reauthorization votes do not help the congressman from getting reelected in the next congressional term, but will be good predictors of the company's next amount of campaign contributions because the past record of donations indicate the company's close association with the congressman i's interest. Note using the previous voting record as the instrumental variable is not possible because even though it might have a positive effect on the amount of campaign contributions, it is also correlated with the current voting behavior of the congressman. Note also that any congressman's characteristics that could determine the amount of campaign contributions affects the congressman's voting behavior. What I then get is the following equation of predicted coefficients:

$$\widehat{CC_i}^t = \sum \widehat{CC_{\{J,i\}}^t}, an$$

$$\widehat{CC^t}_i = \hat{\gamma}_0 + \hat{\gamma}_1 CC_i^{t-1} + \hat{\gamma}_2 prev\_vote_i^{t-1} + \hat{\gamma}_3 party_i^t + \hat{\gamma}_4 fedfunds\_rate_i^t \quad (3)$$

IV. Data

The data on individual campaign donations are provided generously by the Center for Responsive Politics, which include the amount of donations given to each congressman by the four companies starting from 1989. This in turn means that the amount of campaign donations only date back to 1989, so drawing the causality relationship between the campaign donations and the congressional behavior is not possible before 1989. This means the reauthorization votes held before 1989 have to be safely neglected from empirical analysis. Also, for the Roll Call data, a non-governmental information agency called govtrack.us provides up-to-date roll call vote records on every congressional bill. Even though I do not have individual voice vote records, I consider all the incumbents to have voted for the bank's reauthorization if there was a voice vote. If it were the general consensus that the bill was about to pass, the congressional records do not collect individual voting records and place the bill on a voice-vote basis, so I treat them as unanimous. In addition, I do not use the most recent vote held in 2014 because unlike previous reauthorization bills, the 2014 bill was a part of a greater Resolution Bill involving a larger range of clauses. Therefore, the votes are not solely the positions that the congressmen had with respect to the Bank, so are safely neglected.

For the federal funds rate, I retrieved relevant data from the Federal Reserve Bank of New York and calculated the change in the benchmark rate between the voting periods. In other words, I calculated the interest rate differentials between the current voting month and the latest voting month and tracked how the macroeconomy behaved as time progressed. We know if the federal funds rate is high, that means the overall economy is bullish and bearish if otherwise. I attempt to add this variable to see if the amount of campaign donations is affected by macroeconomic vicissitudes, since the heavy manufacturing products sold by the four companies involve high investment and capital and

therefore require bullish economy. Table 1 presents mean summary statistics of campaign contributions received on average by congressmen from each party. Prevcampcon stands for previous campaign contributions received during the previous voting period. Prevcamptotal in turn stands for the total amount of campaign donations received in the previous voting timeframe. This in turn means that except for Independents, the congressmen from both parties received more campaign contributions on average than previous elections every bill cycle since 1992, while the overall amount of campaign donations received increased by a slight margin for Republicans and by a large amount for the Democrats. This suggests that in addition to the four companies I chose, Democrats received overall amount of campaign donations at a much higher than the Republicans as time progressed.

In addition, Table 2 summarizes the voting behavior of congressmen compared to the previous voting behavior. Current vote refers to a congressman's voting behavior at time t and prevvote the same congressman's voting behavior at time t-1. Here the table suggests that the increasing portion of Democrats voted in favor of the Bank's reauthorization. In other words, 14 percent of Democrats that voted no to the bank's reauthorization was reduced to less than 6 percent. Tables 1 and 2 then suggest that during the voting process, there seemed to be a compatible increase in the overall campaign donations received and the increase overall voting behavior of the Democrats.

V. Results

To isolate the effect of campaign donations on congressional behavior on the U.S. Export Import Bank, I attempted to find variables that could act as instrumental variables that could satisfy the conditions for 2SLS regression. For the instrumental variables, I used congressmen's fixed characteristics such as a dummy variable First whether the vote was made during his first time in the office. Also, I used Rank as an instrumental variable that is internally calculated by the PACs for each company. I assumed that there must be some metrics the companies internally use to fund certain congressmen. However, First was neglected as an instrumental variable because it was a part of congressman's fixed characteristics so could be positively correlated with the previous voting behavior. Rank was neglected because the PACs of the four companies involved were not forthcoming with the data. Therefore, since all the spatial variables are inevitably correlated with the overall congressional fixed effects, I considered temporal variables the right candidates for instrumental variables. After using the amount of campaign contributions in the previous voting term, we get Table 2.

Table 1: Summary Statistics of Campaign contributions received from four companies (unit: $)

Party	campcon	currcamptotal	prevcampcon	prevcamptotal
Democrat	10,080	2,179,614	7,877	1,207,740
Independent	12,625	303,535.3	19,000	169,119.5
Republican	12,537	1,291,447	10,434	1,123,533
Mean	11,316	1,713,546	9,191.5	1,163,305

Table 2: Summary Statistics of voting records (1992, 1997, 2002, 2006, 2012 Reauthorization bills)

Party	currentvote	prevvote
Democrat	94%	86%
Independent	100%	50%
Republican	84%	84%
Total	89%	85%

The observations in the two models differ in the numbers because of how some observations have missing previous campaign contributions or the total amount of campaign contributions. The number of observations does make sense because due to the nature of the reauthorization bill that is held in every five years on average, a congressman needs to get reelected to vote again for the bill. So each valid observation in Table 2 should have a congressman's current voting record, his previous voting record (prevvote), campaign donations made by the four companies in that congressional cycle, and those for the cycles before the previous vote (precampcon). If there were several congressional periods when a company donated to a congressman multiple times before the next bill on the bank, then I add the amount to a single number. That will give me the amount of

Table 3: Results using 2SLS and Logit models

	2SLS (1) currentvote	logit (2) currentvote
main		
currentnorm	-0.00153	1.985*
	(-0.85)	(-2.13)
prevvote	0.026	0.537
	(-0.87)	(-1.58)
fedfundsrate	0.0209**	0.133***
	(-2.72)	(-3.57)
nparty	-0.143**	-1.416***
	(-3.02)	(-4.16)
_cons	1.057***	3.418***
	(-24.52)	(-8.23)
N	788	1025

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

$\bar{CC}^t{}_i$ and $\widehat{CC^{t-1}}_i$ so I can then use them to get the coefficients.
coefficients.
After the data cleaning process, I then ran a logit regression according to the model I presented above in equation (1). Table 2 shows the result of the regression of the probability of congressional voting on the bill with the amount of campaign contributions with prevvote that controls congressman's fixed characteristics and fedfundsrate controlling for the macroeconomic variation. Running t-statistics on the coefficients reveals that the coefficient for currentnorm does positively affect the probability of voting at the 5% significance level. We also see a positive influence of

federal funds rate on congressional voting behavior. This is expected because as the economy is bullish, more congressmen would be incentivized to support the Bank so that it could provide lower-than-market interest rates to smaller companies as well as large companies. Also, being a Republican decreases the probability of voting yes by 0.143 for 2SLS methods while the number becomes 0.22 for the logit model. On the other hand, after adding previous campaign donations to the first-stage regression equation, I get a coefficient that is negatively correlated with the current voting behavior, but insignificant at the 5% level, which suggests that the amount of campaign donations the four companies gave do not affect significantly to the overall voting behavior. This is understandable given such a small portion of the overall donations the four companies' donations accounted for. The number will be more significantly determined as more donors that support the bank are identified and added to the pile of donations from parties that side with the bank. What is important to consider, on the other hand, is how private has a smaller magnitude if I treat campaign donations to be endogenous. This in turn suggests that the congressmen's voting behavior vacillates more by the changes in campaign donations that they received after their previous vote than when only previous voting behavior affected the next voting behavior. But the coefficients remain insignificant at the 5% level, so it is hard to compare the results based on their significance.

The results imply two conclusions. First is that macroeconomic considerations prevail over other factors in terms of determining whether the Export-Import Bank should exist for the economy. They indicate that whenever there is a better economy we seem to care for preserving the Bank so that it could address smaller companies that are in need of low interest-rate loans. For how much the campaign contributions are determined and their effect on current congressional behavior, I seem to get a strongly insignificant relation. To summarize, within the scope of my regression analyses congressmen are doing what they think is right for the economy at the time of voting, despite increases in the magnitude of campaign donations they received from both the four companies I added as primary beneficiaries of the U.S. Export-Import Bank and the overall donations from corporate PACs. Second but not least, the signs of the t-statistics and the coefficients do not match, which suggest that the results are not conclusive and the data is prone to error.

VI. Conclusion

The U.S. Export-Import Bank Reauthorization Bill is one of hundreds of bills every congressperson votes for during his or her entire term. Hence, the total amount of contributions one receives is not limited to the companies that are affected by the reauthorization bill. For future attempts to quantify the relationship between money and politics, a more comprehensive knowledge of the beneficiaries of the Bank and the opponents of the Bank that are harmed by such lending activities should be carefully studied. In addition, a more comprehensive study into the actual beneficiaries and the manufacturing competitors should be made. This requires breaking down industries to sub-industry levels to identify who are the most affected by the finance benefits, and this job requires decades of industry analysis to capture the full effects of bank's subsidies.

In addition, since my research focuses only on the hard money, future attempts should tie in both soft money and hard money into campaign finance. While soft money is hard to track due to its clandestine and sensitive nature, it is pivotal to congressmen's campaign finance and therefore should open more doors to the academic world.

THE NATIONAL BASKETBALL ASSOCIATION AND FATIGUE

How Days of Rest Affect NBA Outcome and Game Play

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The nature of the complicated NBA schedule necessitates imbalanced schedules, particularly in regard to days of rest between games. Research in the past shows teams with the rest advantage over their opponent win more often. Currently, both the NBA League Office and individual teams are investing heavily in fatigue analysis and prevention. The League has installed SportVU optical cameras to track player's speeds while teams are using GPS tracking, sleep analysis, and even full body scanners to track the impacts of a demanding schedule. Schedule changes to limit back-to-backs and build in a longer mid-season break attempt to mitigate fatigue, creating the potential a competitive advantage for the better rested team. Using Five Thirty Eight historical schedule data, 2014-15 box scores, and 2014-15 SportVU data, this paper asks if and how fatigue affects both game outcome and in game metrics. I accomplish this using a logit model to predict outcome, means testing to look for differences between 0 rest games and 1+ rest games, and a logit shot making model, all of which incorporate days of rest. This paper finds that both team rest and opponent rest are significant factors in determining game outcome. Searching for in-game effects, this paper learns that scoring, particularly in the paint and on fast breaks, significantly diminish on games of 0 days of rest. Overall, team field goal percentage decreases. Using a shot making predictive models, this paper finds that the shots themselves are not affected significantly by fatigue but rather poorer shot selection and fewer assists are associated with 0 days of rest. The findings suggest teams can now focus on and game plan around the specific aspects of the game that are most affected in hopes of mitigating the effects.

Introduction

The constraints of a National Basketball Association (NBA) season creates unfavorable traveling conditions. Fitting 82 games into a season requires teams to play two games in two days, often which requires overnight travel if the team is to play the second of the back-to-back games on the road. Looking only at games from the 2013-14 season, a startling 96.2% of back-to-backs required overnight traveling (Haberstroh, 2014). One solution may be to shorten the season, yet this coincides with limiting revenue, so it is an unlikely proposal. Similarly, stretching the 82 games over a longer time period would put the NBA at further competition with other majors sports like the National Football League and Major League Baseball as well as shorten the much needed offseason where players both rest and play for their national teams in worldwide competitions. Hence, fatigue will remain a factor in the NBA for years to come.

In the early days of the association, transport from city to city proved difficult. Teams traveled from city to city on mostly trains in order to limit travel expenses. In the very first seasons, the NBA even experimented with a type of revenue sharing system that supported teams who were burdened with an above average travel cost. David Surdam (2012), who wrote The Rise of the National Basketball Association, recounted one story of these expeditions from the 1950s for the New York Knicks, who had just traveled by train to a quiet stop in Indiana:

"The only thing there was an uncovered wooden platform. Traveling with the Knicks, I knew that our instructions were to walk on a two-lane blacktop road toward a blinking yellow light a half-mile away. That turned out to be the only light at a crossroads where there were ten or twelve buildings, nothing taller than two stories. Then we were to look for the plate glass window with the sign of the Green Parrot Café. Carl Braun was our designated shooter of the pebbles up to the second-floor window, a frowsy-haired woman would look out and say, 'Oh the Knicks.' She'd get on the phone, and in a little while four or five cars would gather and drive us the forty miles into Fort Wayne. We'd go right to bed get up for the game that day or night. That was the only way to get to Fort Wayne from Rochester."

The NBA has come a long way since the days of trains, busses and rock throwing; yet an ever busy schedule combined with exogenous variables account for significant player fatigue.

In this paper, I examine the effects of fatigue, defined by the days of rest between games, on both the outcome of the game and game play itself. I first look at the historical NBA schedule, going back to the 1940s, to examine the role fatigue has played in the outcome of games over the decades. Then, using data from the 2014-15 season, I analyze both box score statistics and publicly available SportVU data in an attempt to identify which aspects of game play are most affected by fatigue. Next, I look at history of NBA game outcomes to search whether fatigue is becoming more or less of an advantage. Finally, using SportVU shot log data and Krishna Narsu's shot difficulty metric, I search for which type of player and what type of shot is most affected by fatigue. My hope is that through this analysis, teams are able to plan around the aspects of the game most affected by exhaustion.

The Professional Sports Market and the National Basketball Association

For the National Basketball Association, gameplay and the competitive balance (or lack thereof) is of the utmost importance for both individual teams and the League Offices. Teams strive to improve their roster and coaching to gain a competitive advantage while the League as a whole must compete successfully against other professional leagues to maintain profitability. There are natural barriers to this competitive balance, such as unbalanced markets leading to more revenue for one team than another.

To combat this, the NBA uses methods such as amateur drafts, salary caps, and luxury taxes to an attempt to level the playing field. Competitive balance is necessary to create uncertainty in the outcome of the game, or there would be little incentive to watch and support an inferior team. (McFall, 2014). Despite the tactics of the NBA, there are still artificial competitive imbalances that come from an 82 game schedule. Unbalanced schedules, similar to talent imbalances between Conferences, punish certain teams. Along with this, unlike in the National Football League, teams do not have uniform time to prepare for games . The NBA schedules is equally split between home and away to try to mitigate home court advantage, yet just over 50% of the games in the most recent 2014-2015 season do not offer equal rest, and the resulting win percentages illustrate the advantage (See Table 1 and Table 2 in appendix). Fitting a full season's worth of games into a six month schedule creates travel strains on players, creating an advantage for the better rested team. In the past ten years, franchises have recognized that combatting fatigue can provide an edge and investing in staff, research and improved travel to mitigate the effects.

Law of Increasing Opportunity Cost

Fatigue in the NBA can best be associated with the economic principal of the Law of Increasing Opportunity Cost. Opportunity cost is defined as, "the best alternative that we give up, or forgo, when we make a choice or decision" (Case et al, 2009). The demand for the NBA is as high as ever, as evidenced by unparalleled increases in Basketball Related Income, the tradeoff between maintaining a full schedule and keeping a rested product on the floor must be examined (Draper, 2015). The League must balance the health of players, who are adversely affected by fatigue by increasing the chances of injury or performing below peak levels, both of which limit the earning capacity of a player over a career. So far, the NBA League Office, which builds the schedule, has decided to sacrifice player fatigue in exchange for fewer games. While the League has responded to criticism and reduced certain types of games, such as four games in five days and long haul back-to-backs, the schedule remains treacherous for those expected to play significant minutes in at least 82 games (Partnow, 2015).

General Research on the Effects of Fatigue

Fatigue affects workers in all workplaces. A study conducted by Shahrokh-Shahraki and Nooh –Bin Abu Baker on the effects of fatigue on workplace productivity identified two types of fatigue: physical and nervous. The researchers found that efficiency is linked to fatigue, mild rhythm of the work volume and sufficient sleep. Focusing on sleep deprivation, the study found that this fatigue increases the probability of accidents in taxi drivers as well mental distress, physical disorders, and cardiovascular diseases (Shahraki and Baker, 2011). Extrapolating this to the NBA, sleep deprivation and an inconsistent substitution pattern decrease the efficiency of players.

Investments in Fatigue Mitigation in the NBA

Given the evidence of the effects of fatigue on the general workplace, it is no surprise that NBA teams are racing to understand fatigue own workplace. On a league wide level, the introduction of SportVU optical tracking technology in every arena beginning the 2013-14 NBA season measures the movement of every player on the court 25 times every second. On a team level, organizations are taking advantage of wearable technology offered by companies such as Catapult, an Australian firm that works with roughly 20 NBA teams (up from eight at the beginning of 2013) and countless others in the NFL, English Premier League, and NCAA, among other league around the world. These devices can track force exertion on player movements, further adding more data that can track for anomalies (Lowe, 2010).

Off the court, the Dallas Mavericks began to analyze the sleep patterns of players during the 2013-14 season (Caplan, 2013). In attempts to keep up, the Los Angeles Lakers recently bought a full body scanner from the German company Human Solutions, which historically has worked with the fashion industry (Holmes, 2015). Perhaps most significantly, the most recent NBA champion, the Golden State Warriors, are at the forefront of rest analysis. use data analysis to create a readiness metric for players for each game based on a variety of fatigue related resources ranging from SportVU metrics, heart variability data, and even player surveys (Haberstroh, 2015). The League and teams are hoping this investment helps mitigate injuries related to fatigue.

Literature Review

Prior to delving into my own data, methodology, and results, it is necessary to take an extensive look at previous academic research. Looking first at the NBA schedule, research by Bean and Birge (2010) shows that NBA schedule is in no way optimized, mostly due to the other schedule requirements of an NBA arena and TV slots. Thus,

a fully rest-optimized schedule is not practical given the constraints. Looking particularly at back-to-back game scheduling, research by Kelly (2010) into the back-to-back games finds there is no bias for which teams receive back-to-backs, so there are no geographical factors in play. Looking at fatigue analysis, research by Nutting (2010) shows fewer days of rest and traveling west across time zones are significant advantages while total mileage is insignificant. A similar study by Ashman et al. (2010) finds that betting markets are inefficient in capturing these rest and time-zone advantages. Further research at University of Pennsylvania by Entine and Small (2007) suggests that 10% of the point differential of games can be attributed to rest. All of the previous research focuses on regular season games, yet Silver (2015) finds that teams who play longer playoff series are more likely to lose in the following round. This suggests cumulative effects, for the NBA does not have any back-to-back games in playoff scheduling. Having thoroughly examined the effect of fatigue on outcome, some additional research looks at the in-game effects of fatigue. A paper by Mah et al. (2011) finds sleep extension significantly improves shooting, and research by Yudelman (2015) finds that players take more difficult shots during the second game of a back-to-back.

Data and Methodology

To examine the effects of fatigue, I use historical schedule results and Elo rating from Five Thirty Eight, individual game data, including traditional and SportVU box score statistics, from the 2014-15 season, and finally the individual shot logs for every shot taken in the 2014-2015 season. These data are used to determine whether rest is a significant factor in determining game outcome, which statistics are most affected by rest, and whether shot making is independent of rest. With the exception of the schedule and Elo rating data, all data is pulled from an online database provided by Darryl Blackport, who collected the data from NBA.com/Stats.

I. Schedule And Elo Rating

The historical schedule results is a collection of American professional basketball games, beginning in 1947 all the way up to end of the 2014-15 season. From this sample of 63,157 individual games, I am able to calculate the days of rest between games for each team. The dataset itself includes the location and date of the game, the Elo rating for each team entering and leaving the game, and the final score. Focusing first on the days of rest, the data show how the travel requirements in the NBA have changed for the decade. Looking at Table 3, we see that the 1950s, 1960s, and 1970s had far more back-to-back games than recent decades while recent decades have seen the percentage of back-to-backs stabilize around 23%. Breaking down the schedule data even further into just away games in Table 4, we can see that visitors are forced to play the majority of the 0 rest games in every decade. Amazingly, teams in the 1950s and 1960s played almost half of their road games on 0 rest. Looking to later decades, we can see that the advent of more efficient travel in the 1970s and 1980s led to less demanding travel requirements, so teams would not be forced to play several back-to-back games on one trip because of how much time travel required. However, since the 1990s, there again has been little change.

Given the imbalance of schedules, this paper intends to search to see whether rest has been a significant factor in the outcome of games. The model requires a measure of team quality, so we will use the Elo Rating, a Five Thirty Eight zero sum metric. Elo Rating intends to measure the quality of the team at any given time relative to the rest of the league. It is calculated by using the only the final score of the game, the location of the game, and the Elo Rating of each team coming into the game. Upsets and large score margins lead to a higher Elo increase (Silver and Fischer-Baum, 2015). One disadvantage of depending on the result of the game is that team strength can change suddenly in an offseason or through an in-season trade or injury, and this is not captured by Elo.

To add a further level to the research, the models search for how the effect of rest has changed over time. Each logit model is run for every decade of data available, which ranges from the 1940s to the 2010s. The results will show when and if rest became a significant factor in determining the outcome of games.

$$(1)\ Outcome_{is} = X\beta_{is} + \alpha_1(TeamRest1Day_{is}) + \alpha_2(TeamRest2Days_{is}) + \alpha_3(TeamRest3PlusDays_{is}) + \gamma_1(OpponentRest1Day_{is}) + \gamma_2(OpponenTeamRest2Days_{is}) + \gamma_3(OpponentTeamRest3PlusDays_{is}) + \varepsilon$$

$$(2)\ Outcome_{is} = X\beta_{is} + \alpha(TeamDifferential_{is}) + \varepsilon$$

In both models, Outcome is categorized into wins, assigned as 1, and losses, assigned as 0. X contains controls for team quality (the Elo ratings) and the location of the game. For Model (1) α_n represents the effect of n days of team rest on the win probability while γ_i represents the effect of n days of opponent rest. Days of rest greater than or equal to 3 days are grouped together. The base case for these indicator variables is when a team has no rest between games. Model (2) simply takes (team rest – opponent rest) as the variable of interest for each game. The subscripts i and s indicate the team and date of game.

Looking at the models, the first important question to be asked is if fatigue is a significant factor in determining the outcome of the game. Previous studies have concluded so, yet these data give the most thorough and extensive history of the NBA. Moreover, the extensive data can be partitioned to search for effects over different time periods. To investigate the significance, I use the two models to estimate the effect of fatigue on the outcome of games. The null hypothesis is that any level of team rest and opponent rest have coefficients equal to zero. I anticipate that the null hypothesis will be rejected, for previous literature supports the idea that rest is a significant factor.

Assuming that fatigue is in fact significant for determining the outcome of the game, the next logical question to be asked is how does the effect manifest itself in the actual gameplay. To look at this, this paper used data from game-by-game box score statistics for the 2014-15 regular season is used to search for which statistics are significantly affected by fatigue.

Given that the NBA regular season has 1,230 games, the study looks at 2,460 team-level box scores matched to the corresponding days of rest for the respective team. This data includes traditional box scores, advanced box scores, miscellaneous box score statistics, the four factors, player tracking, and possession type. The intent

is to find which statistics are most significantly affected by fatigue. Using the null hypothesis that the difference in means between 0 days of rest games and 1+ days of rest, I will be able to see which, if any, box score statistics are affected by fatigue.

Following this means testing, the paper looks at how shot making is affected by days of rest. I want to be able to look at this beyond the scope of just field goal percentages, so this paper builds a shot making model that incorporates fatigue to search for significance. Since 2013, the NBA has published SportVU shot logs with detailed data of almost every shot taken. To search for which factors help determine whether a shot will be made, this paper uses 2014-15 shot log data of 202,946 shots. To build my model, I will adopt a version of Krishna Nasru's KOBE shot difficulty metric. Nasru used the SportVU data and concatenated it with player heights to control for height difference. The model's goal is to identify the expected number of points for a shot based on the scenario under which the shot is taken .

The model uses Nasru's model as a base, but adds indicator variables for days of rest. Similarly to the game outcome regressions, this model is a logit regression where the null hypothesis is that all levels of days of rest have a non-significant effect on shot outcome. I anticipate rest will be significant, for the players with more fatigue should have more tired legs. The model is as follows:

$$(3)\ ShotOutcome_{iis} = X\beta_{iis} + \alpha_1(TeamRest1Day_{is}) + \alpha_2(TeamRest2Days_{js}) + \alpha_3(TeamRest3PlusDays_{js}) + \varepsilon$$

In this model, the dependent variable is whether a shot is made or missed. Similar to models (1) and (2), α_i acts as the coefficient for the days of rest for the shooter. Subscript j represents the number shot for player on team i at date s.

To model shot making to search for the effect of the exogenous days of rest, the model controls for the following endogenous variables gathered from the SportVU dataset. Shot distance is the distance the shooter is away from the basket at the time of the shot. Closest defender distance is the distance between the shooter and the closest opposing player. Dribbles are divided into three groups: no dribbles, one dribble, and 2+ dribbles. Location is divided into home and away. Shots are divided into open or contested, defined by whether a defender is within 3.5ft of the shooter. The shot clock is divided into six groups: 24-22 seconds, very early (21.9 to 18 seconds), early (17.9 to 15 seconds), average (14.9 to 7 seconds), late (6.9 to 4 seconds), very late (3.9 to 0 seconds), and none (no shot clock). Touch time is a continuous variable. Rest again is divided into 0 days, 1 day, 2 days, and 3+ days. Finally, the model also includes an interaction term between the shot clock and touch time.

It is also important to note that not every player is equal nor is every shot privy to the same factors, so the data are divided into nine separate groups based on two levels: The height of the player to estimate position and the distance of the shot. The three player groups were defined as guards who are less than or equal to 6 foot 3 inches, wings who are greater than 6 foot 3 inches and smaller than or equal to 6 foot 9 inches, and big men who are taller than 6 foot 9 inches. Shots are then divided up by the distance of the shot from the basket. These categories are less than or equal to 5 feet, greater than 5 feet and less than or equal to 10 feet, and greater than 10 feet. The first group is intended to estimate shots near or at the basket, the second are in between shots in the paint but not at the basket, and the third group is for jump shots

Results

Tables 13 and 14 show the results for the historical season regression (model (1)) using indicator variables. Each regression is split up into the decade of the season in an attempt to see the time effects. Despite having a schedule with many back-to-backs and poor travel conditions, the 1950s showed no fatigue effect for the outcome of games. As travel improved and the number back-to-back games decreased, days of rest became a significant factor for some of the levels tested. However, in recent decades, the different levels of rest have become even more significant at all indicator levels in determining the outcome of the games. This suggests that fatigue is an important factor for the modern day NBA and that the number of days of rest between games is more important that just having 1+ days of rest versus 0 days of rest. Given the null hypothesis that rest does not affect the outcome of games, I can say with great confidence that I can reject the hypothesis. All of the coefficients measuring team rest are positive while all the coefficients measuring opponent rest are negative. This supports conventional theory. However, it is surprising that the 1940 and 1950 model showed little effect.

Tables 15 and 16 (logit results of model (2)) support the findings in Tables 13 and 14. The difference in rest has a positive effect on the outcome of games. However, the difference is only greatly significant in recent decades, particularly 2000 and 2010. This suggests that an edge in rest is becoming more and more of a competitive advantage. This may be due to greater investment in recovery methods and technology. The benefit of one extra day of rest is far greater in recent years than in the past.

One would assume that as travel improved, the effects of fatigue would be mitigated, yet this does not seem to be true. I attest this to the demanding NBA season, where teams are spread about the continental United States, and travel on charter planes is not a perfect substitute for sleeping at home. Moreover, as the NBA is a physical game by nature, the increasing cost of playing games in a short time span lends itself to fatigue; this is a factor where the only solution may be limiting the minutes of players. Travel and recovery improvements can only have a marginal effect for players that are forced to play significant minutes continually over a season.

Given that rest effects are indeed significant, I can look critically at the results of the means testing. Tables 5 through 10 in the Appendix show the results of means testing for 0 days of rest and 1+ days of rest. (Note that some statistics are repeated under different tables). The results are summarized as follows:

- Both offensive and defensive ratings are significantly negatively affected by 0 days of rest. In all other games, the net rating is essentially 0.624 compared to -2.018.
- Team field goal percentage decreases significantly. This is accompanied by fewer points in the paint.
- Teams play at a significantly slower pce

with 0 rest, which manifests itself in fewer fast break points for the offense.

- Ball movement is significantly affected. There are fewer assists and secondary assists, lower assist ratio, and a higher turnover percentage. This is despite an insignificant difference in passes and touches between the two types of games.

- Defense is affected too. There are significantly fewer points off turnovers, fewer steals, and fewer blocks.

Although the data is only from the most recently completed 2014-15 season, we see that ball movement is significantly impaired by fatigue. Interestingly, teams pass at the same rate regardless of rest, yet it is the quality of pass that suffers as represented by the decrease in assists and secondary assists. Quality passing requires split second decision making, so more fatigue seems to slow this reaction time. Similar reaction statistics like steals and blocks also significantly decrease. These results are supported by the previously discussed Shahrokh-Shahraki and Nooh –Bin Abu Baker study regarding taxi drivers and the increase probability of accidents. Moreover, teams are running the same distance in the games, yet with significantly slower pace and fewer fast break points. This suggests effort, or at least willingness to exert excessive effort, is affected. Most importantly to game outcome, however, is the decrease in net rating and field goal percentage. To investigate this thoroughly, we look towards the shot making models.

The results of model (3) seen in Tables 15, 16, and 17 for shot making fail to reject the null hypothesis that rest has no effect. While some of the coefficients for the days of rest are significant, there is no discernable pattern among any of the model groups, so I will attest this minor significance to noise in the data. This is a very surprising result. Conventional thinking suggests that tired legs hurt shot making, yet the models suggest otherwise. Looking at the other variable, the models are consistent in that shot distance, defender distance, height differential, and openness are significant in shot making. For jumpers in particular, taking a dribble and shot clock have negative effects on shot making. This supports the thinking that isolation basketball late in the shot clock is not advantageous for shot making.

Further investigating why field goal percentage and offensive rating decrease with no effects of rest on the actual shots, I performed means testing for certain continuous variables and shot location. These results are available in Tables 11 and 12. The data shows that teams shoot significantly fewer shots at the basket in 0 rest games. Moreover, the shots are taken earlier in the shot clock, after more dribbles, and further away from the basket. This is a key finding. Teams are simply shooting more difficult shots on zero days of rest. The tired legs do not seem to be affecting a player's elevation on a shot or the angle of the arm of the shot. Rather, offenses are taking more difficult shots, which have a lower probability of being successful. Previous preliminary research suggests that this is most significant in two point shots rather than three point shots (Yudelman, 2010). Then, in theory, teams who depend more on three point shots would be less affected by fatigue.

Conclusion

This paper looks at the effects of fatigue, defined as days of rest between games, on the outcome of games, in-game metrics, and shot making. Using two logit models to search for fatigue effects on outcome, I find that fatigue was not a significant factors in the 1950s, but it has been increasingly significant since then. Furthermore, I found that the difference in days of rest for the two teams playing is also significant in more recent decades. Using means testing on box score statistics, I found that ball movement, pace, net rating, defensive movement, and shot selection is significantly hurt by 0 days of rest compared to all other games. Finally, using a shot making metric, I found that shots of similar context are not affected by rest. This suggests that teams on 0 days of rest are struggling to find good shots rather than missing shots the players usually would make.

In future research, I would like to look at the effects of traveling across time zones for back-to-back games. Previous literature suggests that this is significant in determining game outcome, and I would like to see how that affects both the game statistics and shots. Furthermore, I would like to test my hypothesis that teams that rely more on threes are less affected by fatigue. Chart 1 in the appendix looks at total three point field goal attempts in the 2014-15 season and the win percentage in 0 rest games. This can be looked at in a future study that builds an empirical model looking at how reliance on three point shots are related to the ability to compete in games with a fatigue disadvantage.

Appendix

Significance codes for the tables:

"*" = .10 level
"**" = .05 level
"***" = .01 level

Selected tables on the following page.

Table 1: Days of Rest for the 2014-15 NBA Season

		Opponent Days of Rest					
		0	1	2	3+	NA	Grand Total
Team Days Of Rest	0	94	54	18	5		171
	1	222	457	87	18		784
	2	62	72	35	10		179
	3+	27	25	7	20		79
	NA	4				13	17
	Grand Total	409	608	147	53	13	1230

*All days with 3 or more days of rest are grouped as one, and "NA" represents opening day games. Team days of rest are from the home team's perspective. Includes two neutral games.

Table 2: Win Percentage by Days of Rest for the 2014-15 NBA Season

		Opponent Days of Rest					
		0	1	2	3+	NA	Grand Total
Team Days Of Rest	0	53.19%	48.15%	61.11%	80.00%		53.22%
	1	54.95%	58.64%	54.02%	55.56%		57.02%
	2	62.90%	66.67%	45.71%	60.00%		60.89%
	3+	62.96%	60.00%	57.14%	60.00%		60.76%
	NA	50.00%				76.92%	70.59%
	Grand Total	56.23%	58.72%	53.06%	60.38%	76.92%	57.48%

*All days with 3 or more days of rest are grouped as one, and "NA" represents opening day games. Team days of rest are from the home team's perspective. Includes two neutral games.

Table 3: NBA Schedule and Days of Rest

Days of Rest	1940s	1950s	1960s	1970s	1980s	1990s	2000s	2010s	Grand Total
0	25.82%	41.86%	42.06%	34.32%	27.10%	24.46%	22.95%	23.13%	28.60%
1	33.23%	30.97%	32.71%	37.03%	44.70%	50.82%	52.66%	55.06%	45.42%
2	20.42%	16.01%	15.13%	17.43%	18.89%	15.98%	16.20%	14.53%	16.57%
3+	18.89%	9.81%	8.94%	10.08%	8.16%	7.55%	7.04%	6.11%	8.23%
NA	1.64%	1.35%	1.16%	1.14%	1.14%	1.19%	1.14%	1.18%	1.18%

Table 4: NBA Schedule and Visitor Days of Rest

Away Days of Rest	1940s	1950s	1960s	1970s	1980s	1990s	2000s	2010s	Grand Total
0	34.18%	47.52%	49.24%	42.56%	33.94%	32.38%	31.36%	31.30%	36.35%
1	34.18%	32.80%	30.79%	34.60%	41.26%	46.11%	48.19%	50.48%	42.07%
2	16.30%	12.13%	12.23%	14.47%	17.05%	14.31%	13.81%	12.30%	14.17%
3+	13.97%	6.40%	6.76%	7.39%	6.69%	6.07%	5.62%	4.85%	6.36%
NA	1.38%	1.14%	0.98%	0.98%	1.06%	1.13%	1.02%	1.07%	1.05%

Table 5: Means Testing for Advanced Box Scores Statistics

Advanced Box Score Statistic	0 rest	1+ rest	p-value	Significance Code
Offensive Rating	102.301	103.355	0.050	*
Defensive Rating	104.321	102.731	0.003	***
Net Rating	-2.018	0.624	0.000	***
Assist Percentage	0.580	0.589	0.107	
Assist to Turnover	1.646	1.697	0.133	
Assist Ratio	16.549	16.967	0.008	***
Offensive Rebounding Percentage	0.247	0.250	0.367	
Defensive Rebounding Percentage	0.748	0.752	0.307	
Rebound Percentage	0.497	0.501	0.066	*
Team Turnover Percentage	14.820	14.732	0.651	
Efficient Field Goal Percentage	0.493	0.499	0.065	*
True Shooting Percentage	0.532	0.536	0.139	
Usage Percentage	0.199	0.199	0.030	**
Number of Possessions Per Game	95.763	96.480	0.001	***
Player Impact Estimate	0.486	0.504	0.000	***

*

Table 6: Means Testing for Four Factors Statistics

Four Factors Statistics	0 rest	1+ rest	p-value	Significance Code
Efficient Field Goal Percentage	0.493	0.499	0.065	*
Free Throw Rate	0.275	0.277	0.606	
Team Turnover Percentage	0.148	0.147	0.655	
Offensive Rebounding Percentage	0.247	0.250	0.367	
Opponent Efficient Field Goal Percentage	0.499	0.497	0.402	
Opponent Free Throw Rate	0.282	0.275	0.167	
Opponent Team Turnover Percentage	0.144	0.149	0.006	***
Opponent Offensive Rebounding Percentage	0.252	0.248	0.308	

*All numbers are rounded to 3 decimal points

Table 7: Means Testing for Traditional Box Score Statistics

Traditional Box Score Statistic	0 rest	1+ rest	p-value	Significance Code
Field Goals Made	36.934	37.704	0.001	***
Field Goals Attempted	83.188	83.700	0.135	
Field Goal Percentage	0.445	0.452	0.009	***
3 Point Field Goals Made	7.947	7.814	0.401	
3 Point Field Goals Attempted	22.402	22.421	0.952	
3 Point Field Goal Percentage	0.353	0.346	0.171	
Free Throws Made	17.066	17.153	0.762	
Free Throws Attempted	22.513	22.934	0.235	
Free Throw Percentage	0.758	0.748	0.038	**
Offensive Rebounds	10.744	10.931	0.301	
Defensive Rebounds	32.014	32.539	0.032	**
Rebounds	42.758	43.470	0.018	**
Assists	21.440	22.216	0.001	***
Steals	7.494	7.810	0.026	**
Blocks	4.497	4.888	0.001	***
Turnovers	13.651	13.725	0.702	
Personal Fouls	20.449	20.134	0.138	
Points	98.881	100.375	0.008	***
Point Margin	-1.620	0.500	0.009	***

*All numbers are rounded to 3 decimal points

Table 8: Means Testing for Player Tracking Statistics

Player Tracking Statistic	0 rest	1+ rest	p-value	Significance Code
Distance	16.623	16.667	0.189	
Offensive Rebounding Chances	20.484	20.785	0.314	
Defensive Rebounding Chances	51.451	51.810	0.388	
Rebounding Chances	71.934	72.595	0.193	
Touches	422.055	422.606	0.760	
Secondary Assists	5.174	5.489	0.009	***
Free Throw Assists	2.212	2.265	0.476	
Passes	301.439	301.280	0.967	
Assist	21.440	22.216	0.001	***
Contested Field Goals Made	20.865	21.527	0.001	***
Contested Field Goals Attempted	46.257	46.423	0.065	*
Contested Field Goal Percentage	0.454	0.466	0.000	***
Uncontested Field Goals Made	16.067	16.174	0.597	
Uncontested Field Goals Attempted	36.926	37.271	0.268	
Uncontested Field Goal Percentage	0.435	0.434	0.756	
Field Goal Percentage	0.445	0.452	0.009	***
Defended Field Goals Made	15.998	16.080	0.744	
Defended Field Goals Attempted	30.718	31.249	0.223	
Defended Field Goal Percentage	0.528	0.521	0.213	

*All numbers are rounded to 3 decimal points

Table 9: Means Testing for Miscellaneous Box Score Statistics

Miscellaneous Box Score Statistic	0 rest	1+ rest	p-value	Significance Code
Points off Turnovers	15.508	16.356	0.003	***
2nd Chance Points	13.038	13.299	0.298	
Fast Break Points	12.307	13.357	0.000	***
Points in the Paint	40.370	42.580	0.000	***
Opponent Points off Turnovers	16.406	16.078	0.260	
Opponent 2nd Chance Points	13.200	13.249	0.848	
Opponent Fast Break Points	13.240	13.069	0.576	
Opponent Points in the Paint	42.238	42.003	0.590	
Blocks	4.497	4.888	0.001	***
Block Attempts	5.016	4.728	0.021	**
Personal Fouls	20.449	20.134	0.138	
Personal Fouls Drawn	20.003	20.272	0.199	

*All numbers are rounded to 3 decimal points

Table 10: Means Testing for Possession Statistics

Possessions Statistic	0 rest	1+ rest	p-value	Significance Code
Touches	422.078	422.671	0.742	
Front Court Touches	310.285	310.213	0.966	
Elbow Touches	18.147	18.458	0.372	
Post Touches	17.112	17.970	0.001	***
Paint Touches	13.598	14.454	0.000	***

*All numbers are rounded to 3 decimal points

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